V International Symposium and Workshop on Hamiltonian Systems and Celestial Mechanics

HAMSYS-2008, July 7-11, 2008

CIMAT, Guanajuato

Abstracts

Acosta–Humánez, Primitivo Belén

The Spring Pendulum system via Morales-Ramis theory using Kovacic Algorithm

Universitat Politécnica de Catalunya, SPAIN
primitivo.acosta@upc.edu

Morales-Ramis theory is the Galois theory in the context of dynamical systems. In particular is the best approach known to detect non-integrability of hamiltonian systems through the differential Galois groups corresponding to its normal variational equations.

Kovacic algorithm is an algorithm to solve second order linear differential equations with rational coefficients such normal variational equations of hamiltonian systems with two degree of freedom.

In this talk we show the non-integrability of the Spring Pendulum hamiltonian system through Morales-Ramis theory using Kovacic Algorithm. The non-integrability of the spring pendulum system has been analyzed in [1.] via Ziglin theory, (i.e. through monodromy groups) and also has been analyzed in [2.] via Morales-Ramis theory. In both cases has been avoided the Kovacic algorithm.

References

[1] R. Churchill, J. Delgado and D. Rodd. The Spring-Pendulum System and the Riemann Equation. New Trends for Hamiltonian Systems and Celestial Mechanics, E.A.Lacomba, J.Llibre, Ed., World Scientific, Singapore, 1993.

[2] J. Morales-Ruiz. Differential Galois Theory and Non-Integrability of Hamiltonian Systems, Birkhauser, Basel, 1999.

Almeida Santos, Alan; Alvarez–Ramírez, Martha[(1)] and Vidal, Claudio[(2)]

Central configurations in the $1 + 4$ co-spherical body problem

[(1)] *Departamento de Matemáticas, UAM–Iztapalapa, MEX*
[(2)] *Departamento de Matemática, Universidad del Bío–Bío, CHILE*
[(1)] mar@xanum.uam.mx [(2)] clvidal@ubiobio.cl

In this talk, we will speak about the study central configurations (c.c.) of a special configuration of the spatial 5-body problem, where four masses m_1, m_2, m_3 and m_4 lie on a common sphere, and the fifth one, m_0 lies at the center of the sphere, namely, $1 + 4$ *co-spherical body problem*

Brandão Dias, Lúcia de Fátima

The elliptic restricted three-body problem with collision

Departamento de Matemática
Universidade Federal de Sergipe–Itabaiana, BRA
`luciafadf@hotmail.com`

The elliptic restricted three-body problem with collision is a restricted three-body problem where the primaries move in oscillatory Keplerian motion on a straigh line and the infinitesimal mass is moving in a three dimensional space. We study two particular cases of this problem: when the infinitesimal particle is moving in the perpendicular plane to the primaries motion that passes through the center of mass of the primary system and when the particle infinitesimal is moving in a plane that contains the line of motion of the primaries. We called, respectively, these subproblems by elliptic isosceles restricted three-body problem with collision and elliptic planar restricted three-body problem with collision. Our purpose is to prove the existence of many families of periodic solutions using Continuations method, where the perturbing parameter is related with the energy of the primaries. This work is merely analytic and uses symmetry conditions and appropriate coordinates.

Celli, Martin(1)**; Ernesto A. Lacomba**(2) **and Ernesto Pérez-Chavela**(3)

Polygonal relative equilibria in the N-vortex problem

Departamento de Matemáticas
UAM–Iztapalapa, MEX
`cell@xanum.uam.mx` `lace@xanum.uam.mx` `epc@xanum.uam.mx`

The N-vortex problem consists in the study of the solutions of the equations of Helmholtz which describe the motion of a planar incompressible fluid. This problem presents several analogies with the N-body problem. Some degeneracy of the potential makes some computations easier, but we have to take into account positive and negative vorticities (vorticity is the analogous of mass).

We are interested in relative equilibria made of regular polygons. We show that a relative equilibrium with one polygon with more than three vortices requires equal vorticities. This result is the analogous of Perko-Walter's one in celestial mechanics. Here, the particular case with zero total vorticity adds new difficulties.

We compute the relative equilibria with two polygons and the same vorticity on each polygon. We also study the corresponding restricted problem. This adapts and generalizes the results of Bang-Elmabsout (restricted problem for one polygon in celestial mechanics), Moeckel-Simó (two polygons in celestial mechanics), Aref-Van Buren (two polygons with the same vorticity for all the vortices).

Cors, Josep M.[(1)]; **Barrabés, Esther**[(2)]; **Pinyol, Conxita**[(3)] **and Soler, Josep**[(4)]

Hip–Hop solutions of the $2N$–Body problem with large eccentricity

[(1)] *Departament de Matemàtica Aplicada III, Universitat Politècnica de Catalunya*
[(2)] *Departament d'Informàtica i Matemàtica Aplicada, Universitat de Girona.*
[(3)] *Departament d'Economia i Història Econòmica, Universitat Autònoma de Barcelona*
[(4)] *Departament d'Informàtica i Matemàtica Aplicada, Universitat de Girona*[iv]*, SPAIN*
[(4)] cors@epsem.upc.edu

Hip–Hop solutions of the $2N$–body problem with equal masses are shown to exist using a topological argument. These solutions are close to a planar regular $2N$–gon homographic configuration with values of the eccentricity close to 1, plus a small vertical oscillations in which each mass.

Introduction

We are interested in hip-hop orbits, which are solutions of the $2N$-body problem with equal masses such that all bodies stay for all time on the vertices of an anti-prism (see [6], [1]). The $2N$ bodies can be arranged in two groups of N, each group moving on a rotating regular N–gon configuration in a plane perpendicular to a given axis of the anti-prism. The motion of each group oscillate along this axis, and coincide with opposite velocities at regular intervals in the same plane. The orthogonal projection of both N–gons on the plane perpendicular to the axis of symmetry is always a regular rotating $2N$–gon.

Some results related to hip-hop solutions were obtained mostly by means of variational methods, which make it possible to find solutions that do not depend on a small parameter ([4], [3]). In [1], the authors show that Poincare's argument of analytic continuation can be used to add vertical oscillations to the circular motion of $2N$ bodies of equal mass occupying the vertices of a regular $2N$-gon, and prove the existence of families of hip-hop solutions with eccentricity close to zero.

The aim of this work is to prove the existence of hip-hop solutions with high values of the eccentricity and close to 1. In this case, the analytical continuation method cannot be used, and has been substituted by a topological reasoning. The details can be found in [2].

Main result

Let us consider a suitable reference system of coordinates (x, y, z) such that the two N-gon are in planes perpendicular to the vertical z axis. Let $\mathbf{r}$ be the position of one body. The position (and velocity) of the other $2N-1$ bodies are given by $R^{i-1}\mathbf{r}$, for $i = 2, \cdots, 2N$, where R is a rotation plus a reflection. The problem can be reduced to a system with three degrees of freedom (see [1]) given by

$$\ddot{\mathbf{r}} = \sum_{k=1}^{2N-1} \frac{(R^k - I)\mathbf{r}}{|(R^k - I)\mathbf{r}|^3}. \tag{1}$$

We introduce cylindrical coordinates (r, ϕ, d), and their corresponding momenta (p_r, p_ϕ, p_d). The Hamiltonian of the problem does not depend on ϕ, which means that $\dot{p_\phi} = 0$. We fix the angular momentum $p_\phi = \Phi$, and we consider the *reduced problem* in a rotational frame given by the equations

$$\begin{aligned} \ddot{r} &= \frac{\Phi^2}{r^3} - 2r \sum_{k=1}^{2N-1} \frac{\sin^2\left(\frac{k\pi}{2N}\right)}{\left(4r^2\sin^2\left(\frac{k\pi}{2N}\right) + ((-1)^k-1)^2 d^2\right)^{3/2}}, \\ \ddot{d} &= -\frac{d}{2} \sum_{k=1}^{2N-1} \frac{\left((-1)^k-1\right)^2}{\left(4r^2\sin^2\left(\frac{k\pi}{2N}\right) + ((-1)^k-1)^2 d^2\right)^{3/2}}. \end{aligned} \tag{2}$$

Notice that the initial conditions $d(0) = \dot{d}(0) = 0$ will result in a planar (in the $z = 0$ plane) homographic motion of the $2N$ bodies. We want to add a small vertical motion (in the z direction) and show the existence of periodic 3-dimensional hip-hop solutions. If the vertical motion is small enough, the motion can be decoupled into a planar plus a vertical motion system. The uncoupling can be accomplished through a rescaling of the variable d. Substituting d by εd into the Eq. (2) we obtain

$$\ddot{r} = \frac{\Phi^2}{r^3} - \frac{K_N^2}{r^2} + O(\epsilon^2), \qquad \ddot{d} = -\frac{S_N^2}{r^3}\, d + O(\epsilon^2), \tag{3}$$

where $K_N^2 = \frac{1}{4}\sum_{k=1}^{2N-1} \sin^{-1}\left(\frac{k\pi}{2N}\right)$, $\quad S_N^2 = \frac{1}{64}\sum_{k=1}^{2N-1}((-1)^k-1)^4 \sin^{-3}\left(\frac{k\pi}{2N}\right)$. For $\epsilon = 0$, the system (3) uncouples into a planar Kepler motion plus a vertical oscillation. Using similar techniques as in [5], the existence of symmetric periodic solutions of the problem given by Eq. (3) can be shown for a sequence of values of the eccentricity tending to 1. The main result is

Theorem

Let N, m be positive integers, $N \geq 2$. Consider a homographic motion of a system of $2N$ bodies of equal mass of semimajor axis a^*. There exists an increasing infinite sequence of eccentricities $e_{n,m}$, $n \geq 1$, converging to 1, and an infinite sequence of positive numbers $\epsilon_{n,m}$ such that, for any $\epsilon \in (0, \epsilon_{n,m}]$ there exist a_ϵ and e_ϵ in such a way that $a_\epsilon \to a^*$, $e_\epsilon \to e_{n,m}$ and the solution of system (3) with initial conditions $r(0) = a_\epsilon(1-e_\epsilon)$, $\dot{r}(0) = 0$, $d(0) = 0$, $\dot{d}(0) = \epsilon$ is a Hip-Hop solution of period $T^* = 2m\frac{\pi a^{*3/2}}{K_N}$ of the reduced problem (2).

References

[1] E. Barrabés, J.M. Cors, C. Pinyol, and J. Soler. Hip-hop solutions of the $2n$-body problem. *Celestial Mechanics and Dynamical Astronomy*, 95(1-4):55–66, 2006.

[2] E. Barrabés, J.M. Cors, C. Pinyol, and J. Soler. Highly eccentric Hip–Hop solutions of the $2N$–Body problem. *preprint*.

[3] A. Chenciner and J. Féjoz. L'équation aux variations verticales d'un équilibre relatif comme source de nouvelles solutions périodiques du problème des N corps. *C. R. Math. Acad. Sci. Paris*, 340(8):593–598, 2005.

[4] A. Chenciner and A. Venturelli. Minima de l'intégrale d'action du problème newtonien de 4 corps de masses égales dans $\mathbf{R}^3$: orbites "hip-hop". *Celestial Mech. Dynam. Astronom.*, 77(2):139–152 (2001), 2000.

[5] J. Llibre and R. Ortega. On the families of periodic orbits of the sitnikov problem. *Siam J. Applied Dynamical Systems*, 2008.

[6] S. Terracini and A. Venturelli. Symmetric trajectories for the $2N$-body problem with equal masses. *Arch. Ration. Mech. Anal.*, 184(3):465–493, 2007.

Davila–Rascón, Guillermo[1] **and Vorobiev, Yuri. M.**[2]

The first step normalization for Hamiltonian systems with two degrees of freedom over orbit cylinders

[1,2] *Departamento de Matemáticas, Universidad de Sonora, MX*
[1] davila@gauss.mat.uson.mx

We study a class of perturbed Hamiltonian systems with two degrees of freedom on a phase space equipped with a symplectic form which depends non-uniformly on a small parameter. Our main purpose is to search for a symplectic mapping, smoothly depending on the parameter, which transforms a system of this kind to a nearly integrable system on a canonical model phase space.

Delgado, Joaquín

Heteroclinic connections in the restricted parabolic 3–body problem

Departamento de Matemáticas
UAM–Iztapalapa, MX
jdf@xanum.uam.mx

Let two finite masses m_1, m_2 move along parabolic orbits. A third massless particle is acted upon gravitational attraction of the primaries. In a rotating–pulsating frame where the primaries remain fixed the equations of motion of the infinitesimal takes the form

$$\begin{aligned}\frac{d\theta}{ds} &= \cos\theta,\\ \frac{d\zeta}{ds} &= \beta,\\ \frac{d\beta}{ds} &= -\left(\sin\theta - 4i\cos\theta\right)\beta + \nabla\Omega\end{aligned} \tag{4}$$

where

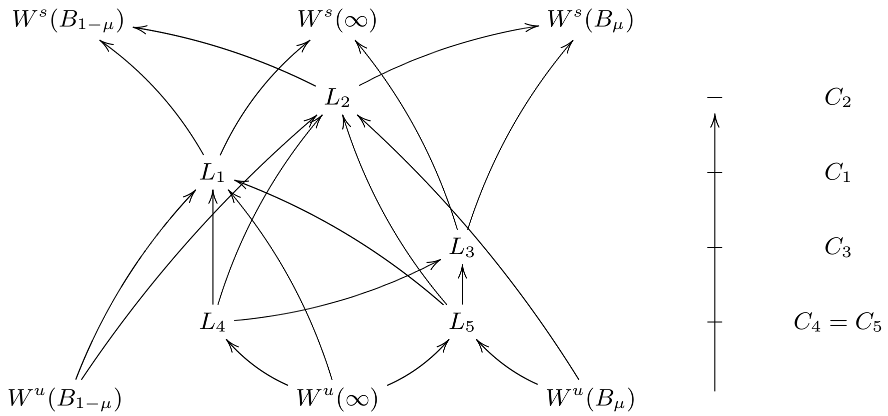
$$\Omega = |\zeta|^2 + \frac{2\mu}{|\zeta+1-\mu|} + \frac{2(1-\mu)}{|\zeta-\mu|},$$

and $\theta \in (-\pi/2, \pi/2)$ corresponds to the original system. The invariant boundaries $\theta = \pm\pi/2$ correspond to final evolutions for $t \to \pm\infty$. Thus the system is defined in the extended phase space

$$\mathcal{H} = [-\pi/2, \pi/2] \times (\mathbb{C} \setminus \{-(1-\mu), \mu\}) \times \mathbb{C}. \tag{5}$$

Al the critical points lie on the invariant boundaries and there are exactly five of them named L_i, $i = 1, 2, \ldots, 5$ as in the circular restricted problem. The full dynamics on the invariant boundaries is studied. Since there exists a Lyapunov function, for a complete description of the dynamics its enough to determine the connections among the critical points. We prove there exists exactly three heteroclinic connections of the type $L_4 \to L_1$, $L_4 \to L_2$, $L_4 \to L_3$. This completes previously connections obtained by the author and Martha Alvarez and Josep Cors.

Theorem. For any value of the mass parameter $\mu \in (0, 1/2)$ the following connections occur:



Here $W^{s,u}(B_\alpha)$ denotes the stable (unstable) set of solutions ending (starting) in double collision with the primary of mass α, and similarly $W^{s,u}(\infty)$ is the set of solutions where the infinitesimal escapes to infinity in forward (backward) time in the rotating-pulsating frame. Dynamical consequences on the existence of final evolutions are derived from this result.

Florin Diacu[(1)], Ernesto Pérez-Chavela[(2)], and Manuel Santoprete[(3)]

The n-body problem in spaces of constant curvature, I

[(1)] *Pacific Institute for the Mathematical Sciences and Department of Mathematics and Statistics University of Victoria, CA*
[(2)] *Departamento de Matemáticas, UAM–Iztapalapa, MEX*
[(3)] *Department of Mathematics, Wilfrid Laurier University, CA*
[(1)]`diacu@math.uvic.ca` [(2)]`epc@xanum.uam.mx` [(3)]`msantopr@wlu.ca`

In this work we generalize the Newtonian n-body problem to spaces of constant curvature, κ, and study the motion in the 2-dimensional case. We derive the equations of motion with the help of the variational methods of constrained Lagrangian dynamics. For $\kappa > 0$, the motion takes place on spheres. For $\kappa < 0$, we use the Weierstrass hyperboloidal model of hyperbolic geometry. This model puts into the evidence two kinds of rotations: circular and hyperbolic. The latter raise some interesting questions and properties. Our results also shed some new light on the classical n-body problem in Euclidean space.

Falconi, Manuel[(1)]; Lacomba, Ernesto A.[(2)] and Vidal, Claudio [(3)]

Classical Mechanical Systems with Homogeneous Polynomial Potential of degree 4

[(1)]*Facultad de Ciencias, UNAM, MEX*
[(2)]*Departamento de Matemáticas, UAM–Iztapalapa, MEX*
[(3)]*Departamento de Matemática, Universidad del Bío–Bío, CHI*
[(1)]`falconi@servidor.unam.mx`[(2)]`lace@xanum.ua.mx`[(3)]`clvidal@ubiobio.cl`

In this talk it will be shown some advances on our project about the dynamical

properties of Homogeneous Hamiltonian Systems where we are mainly focused in the problem to determine how the dynamical behaviour depends on the homogeneity degree. Although at the present, we are only able to describe some dynamical features for degree 3 and 4, these are useful to build some conjectures about the general problem.

Fujiwara, Toshiaki

Quartic curve does not support the figure-eight solution under any homogeneous potential

Kitasato University, JAPAN
`fujiwara@clas.kitasato-u.ac.jp`

The figure-eight solution of the three-body problem under homogeneous potentials including the Newtonian was found by Moore, Chenciner and Montgomery. In this solution, three bodies with equal mass chase each other on eight shaped curve. Simó numerically proved that the eight shaped curve for the Newtonian figure-eight solution is not the algebraic curve of order 4, 6, nor 8. On the other hand, Fujiwara, Fukuda and Ozaki found that a figure-eight motion on the lemniscate, quartic curve $(x^2+y^2)^2 = x^2 - y^2$, satisfies the equation of motion under the inhomogeneous potential $\sum_{i<j}(1/2\ln r_{ij} - \sqrt{3}/24r_{ij}^2)$.

Then, a question arises. Is there a figure-eight motion on some quartic curve that satisfies the equation of motion under a homogeneous potential $-\alpha^{-1}\sum_{i<j} 1/r^\alpha$ with $\alpha \in \mathbb{R}, \alpha \neq 0$ or $\sum_{i<j}\log r_{ij}$? In other words, does quartic curve support figure-eight solution under homogeneous potential?

In this symposium, we will show that (1) quartic curve that may support the figure-eight solution is only the lemniscate $(x^2+y^2)^2 = x^2-y^2$ and its scale variant $x \to \lambda x$, $y \to \mu y$ with λ and μ are constants. This is a quite general result. Indeed, we need only invariance under the translation, rotation, time reverse, and exchange of three-bodies. (2) The lemniscate and its scale variant does not support the figure-eight motion under any homogeneous potential $-\alpha^{-1}\sum_{i<j} 1/r_{ij}^\alpha$ or $\sum_{i<j}\log r_{ij}$. (3) Therefore, the answer of the question is, quartic curve does not support figure-eight solution under any homogeneous potential.

Goes, Eduardo Leandro

Bifurcations and Counting of Central Configuratons in some Restricted N-Body Problems

Departamento de Matemática,
Universidade Federal de Pernambuco, BRA
`eduardo@dmat.ufpe.br`

We present analytical results on the bifurcation set of central configurations in the planar restricted four-body problem. These results confirm the numerical ones obtained by P. Pedersen, in the 1940s, and C. Sim, in the 1970s, and allow us to determine the possible number of central configurations for an arbitrary triple of primaries. The proofs are based on classical analysis, Mbius transformations and Decartes' rule of signs. On a different restricted problem, with an infinitesimal mass and N equal masses forming a Dziobek configuration, we count central configurations using a completely different approach.

De la Llave, Rafael[1] and Gidea, Marian[2]

Perturbations of geodesic flows producing unbounded growth of energy

[1] *University of Texas at Austin, USA*
[2] *Northeastern Illinois University, USA*
[1] llave@math.utexas.edu [2] mgidea@neiu.edu

We present a simplified proof of a result, obtained by Delshams, de la Llave and Seara, stating that for a generic Riemannian metric on a manifold, a quasi-periodic (time-dependent) perturbation with a generic external potential will produce some orbits whose energy grows unboundedly in time. This is in contrast to time-independent perturbations, where the energy is conserved. This result implies Mathers acceleration theorem and is related to Arnolds diffusion problem. We also present a generalization of this result in which the perturbation is not needed to be quasi-periodic but is required to satisfy some recurrence condition.

Hernández, Antonio

Reduction of the N-vortex problem and hamiltonian structure in internal variables

Instituto de Investigación en Matemáticas Aplicadas–UNAM, MEX
antoniohg@mac.com

We will discuss the N-vortex problem on the plane, describing a) the equations of motion in the space of squared distances and the associated hamiltonian formulation in "internal" variables; b) the Poisson and symplectic reduction of the N-vortex problem. As a particular case we will describe the reduction of the four-vortex problem with identical vorticities and its analysis regarding it as a perturbation of the three-vortex problem.

Iturriaga, Renato

Periodic solutions to a nonlinear Schrödinger equation with periodic magnetic field

CIMAT, MX
renato@cimat.mx

We study a nonlinear equation of the form

$$(\wp_A) \qquad (-i\nabla + A)^2 u + Vu = |u|^{p-2}u,$$

where $V : \mathbb{R}^N \to \mathbb{R}$ is the electric potential and $A : \mathbb{R}^N \to \mathbb{R}^N$ is a magnetic potential associated to an external magnetic field B, that is, $\mathrm{curl}A = B$. We assume that $p \in (2, 2^*)$ where $2^* := \frac{2N}{N-2}$ is the critical Sobolev exponent. We also assume that A and V are 2π-periodic in each variable $x_1, ..., x_N$ and look for 2π-periodic solutions $u : \mathbb{R}^N \to \mathbb{C}$. Unlike the nonperiodic case where problem $(\wp_A)$ is essentially unique due to the gauge invariance, in the periodic case two magnetic potentials $A, \widetilde{A}$ with $\mathrm{curl}A = \mathrm{curl}\widetilde{A} = B$ may give rise to different variational principles for the same physical problem. We show that, under appropriate assumptions, $(\wp_A)$ has infinitely many 2π-periodic solutions. We also show that 2π-periodic solutions of $(\wp_A)$ are, in general, not equivalent (in an appropriate sense) to those of $(\wp_{\widetilde{A}})$. Our results are partially inspired by Aubry-Mather theory and complement some recent work of Evans.

Marchesin, Marcelo Domingos

Time-periodic Hamiltonian perturbation of Hamiltonian systems

Departamento de Matemática
Universidade Federal de Minas Gerais, BRA
mdm@dmat.ufpe.br

The result we will show at the Hamsys-2001 concerns the study of time-periodic Hamiltonian perturbation of Hamiltonian systems. Based on the Melnikov Method we will present a simple criteria for determining the stability of the sub-harmonic orbits whose existence is garanted (under certain conditions) by the Melnikov theorem. Such a criteria is mainly based upon the study of the local monotonicity of the Melnikov function corresponding to a specific sub-harmonic orbit of the non-perturbed system. After we present the general case, we show an application to the Sitnikov problem which was the problem that originated such research and about which we presented partial results at the Hamsys-98 in Pátzcuaro.

Meiss, James D.

Generating Forms and Flux for Volume Preserving Maps

Department of Applied Mathematics
University of Colorado at Boulder, USA
jdm@boulder.colorado.edu

Just as one can generate an exact, symplectic map by an implicit generating function, exact volume-preserving maps can be obtained by implicit generating forms. Hector Lomelí and I have have shown recently how to generalize this idea, originally due to Carroll, to obtain generating forms of different "types" analogous to the four standard canonical generators.

A first step in a theory of transport is computation of the flux though lobes of turnstiles. For the area preserving case, these fluxes are given by differences between the actions of the heteroclinic orbits that bound the lobes. We show how to generalize this to the exact volume-preserving case. In this case the generating forms give rise to an "action" given by an integral along heteroclinic curves.

Muciño-Raymundo, Jesús

Vector fields from polynomial maps with one–dimensional fibers

Instituto de Matemáticas–UNAM
Unidad Morelia, MEX
muciray@matmor.unam.mx

Consider a real or complex polynomial map $(F_2, \ldots, F_n) : \mathbb{K}^n \to \mathbb{K}^{n-1}$. There exists a canonical vector field X tangent to its fibers. It generalizes the Hamiltonian vector field in $n = 2$. The dynamics and geometry of such vector fields is described. In particular, using X we study the question of the existence of an additional polynomial F_1, such that the complete map $(F_1, F_2, \ldots, F_n)$ is a local diffeomorphism.

Offin, Daniel

Some new periodic solutions of the three body problem

Department of Mathematics,
Queens University, Kingston, CA
`offind@mast.queensu.ca`

Stability analysis for the periodic hip hop orbits, in the equal mass four body problem.

We shall give a discussion of the technique of stability analysis, using the Conley–Zehnder–Maslov index for periodic orbits in Hamiltonian systems. We will follow this with discussion of two examples: resonance tongues in planar Hamiltonian systems, and the Hip Hop orbits of the equal mass four body problem.

Piña, Eduardo

The form sphere of the gravitational three body problem in the plane

Departmento de Física
UAM–Iztapalapa, MEX
`pge@xanum.uam.mx`

The gravitational three body problem in the plane is studied in the Piña and Jiménez system of coordinates. The form sphere is explicitly computed for three different masses and it is shown it is actually a double sphere. In this sphere the central configurations correspond to fixed points of this sphere. Contrary to other authors the poles of the sphere occur for two equal inertia moments, not for the Lagrange points. The binary collisions are also represented in this sphere. The Moon-Earth-Sun plane problem moves on a small circle of this sphere. The Janus-Epimetheus-Saturn motion on this sphere has a periodic behaviour with the 4 year near collision period.

Prada, Ingmar[1] **and Jiménez–Lara, Lidia**[2]

The three-body problem, symmetries and periodic orbits

[1,2]*Departamento de Física*
Universidad Autónoma Metropolitana–Iztapalapa, MX
[1]`ingmar.prada@gmail.com` [2]`lidia@xanum.uam.mx`

We use the coordinate system introduced by Piña for studying the three-body problem. These coordinates are defined by the principal axes of inertia system (determined by the Euler angles) and three other variables: R_1 and R_2 related with the inertia principal moments, and an auxiliar angle σ. We connect these coordinates with the shape sphere of similaritiy class of triangles and we give a clear geometric interpretation of the angle σ. We then consider the planar three-body problem and its symmetries, and we work with a method that let us find periodic orbits for this problem.

Roberts, Gareth E.

Using BKK Theory in Restricted N-Body Problems

Department of Applied Mathematics
University of Colorado, USA
`gareth@babbage.colorado.edu, groberts@radius.holycross.edu`

We analytically prove Saari's conjecture adapted to the planar, circular, restricted three-body problem (PCR3BP) using Bernstein-Khovanskii-Kushnirenko (BKK) theory. Specifically, we show that it is not possible for a solution of the PCR3BP to travel along a level curve of the amended potential without being fixed at one of the five libration points. Equivalently, the only solutions with constant speed are equilibria. We also show that the number of equilibria in the PCR4BP is finite for any choice of masses. Although hardly surprising, the proof is a nice application of techniques from computational algebraic geometry, including BKK Theory and Gröbner bases.

Rebollo–Perdomo, Salomón

Limit cycles of perturbed Hamiltonian systems of Liénard type

Departamento de Matemáticas
UAM–Iztapalapa, MEX
srebollo@xanum.uam.mx

The study of planar perturbed Hamiltonian systems is a classic idea. One of the simplest planar Hamiltonian systems is the harmonic oscillator. Special polynomial perturbations of this Hamiltonian system give polynomial planar systems of Liénard type. Some limit cycles of such systems are generated, under the perturbation, from cycles of the harmonic oscillator. We will give an upper bound of the number of these limit cycles. Such upper bound depends only on the degree of the perturbation.

Robinson, Clark

Relationship between Melnikov and Subharmonic Melnikov Function

Department of Mathematics, Northwestern University, USA
clark@math.nwu.edu

A topological method is used to prove the existence of diffusing orbits for a Hamiltonian system that shadow transition chains of invariant tori connected by either heteroclinic orbits or Birkhoff zones of instabilities.

Diacu, Florin [1]**; Pérez-Chavela, Ernesto** [2] **and Santoprete, Manuel** [3]

The n-body problem in spaces of constant curvature, II

[1] *Pacific Institute for the Mathematical Sciences and Department of Mathematics and Statistics University of Victoria, CA*
[2] *Departamento de Matemáticas, UAM–Iztapalapa, MEX*
[3] *Department of Mathematics, Wilfrid Laurier University, CA*
[1]diacu@math.uvic.ca [2]epc@xanum.uam.mx [3]msantopr@wlu.ca

This talk will be based on joint work with Florin Diacu and Ernesto Pérez-Chavela. It concerns a generalized Newtonian n-body problem in two-dimensional spaces of constant curvature. We will illustrate several results concerning central configurations and we will prove Saari's conjecture for "moving geodesics".

Delshams, A.[1], **Kaloshin, V.**[2] **and** **Seara, T.M**[3]

Arnold diffusion along nearly parabolic orbits for the planar restricted planar 3-body problem Abstract

[1,3] *Departament de Matematica Aplicada I*
Universitat Politécnica de Catalunya, ES
[3] Tere.M-Seara@upc.edu

We consider the (elliptic) restricted planar three body problem (RP3BP), with the primaries moving in a elliptic orbit of small but fixed eccentricity about their center of mass. Using geometrical methods, we prove the existence of Arnold diffusion along nearly parabolic orbits. Concretely, we see that when the mass ratio between the primaries is small enough, the angular momentum of the comet can overcome big changes.

De la Llave[1]**, Rafael; Fontich, Ernest**[2] **and** **Sire, Yannick**[3]

Construction of whiskered tori for maps and flows in finite and infinite systems.

[1] *University of Texas at Austin, USA*
[2] *Universitat de Barcelona, SP*
[3] *Université de Marseille, FR*
sire@cmi.univ-mrs.fr

We develop a general “a posteriori” KAM theory which allows to construct whiskered tori for maps and flows in finite dimensions. This type of method is also used to construct finite dimensional tori for coupled maps on lattices, i.e. in an infinite dimensional framework. We never assume that the system is written in action-angle coordinates and that the stable and unstable bundles are trivial. The method also allows to construct infinite dimensional tori (almost-periodic solutions) on lattices thanks to a clustering argument.

Stoica, Cristina

Constant inertia trajectories: on Saari’s conjecture and more

Department of Mathematics
Wilfrid Laurier University, CA
cstoica@wlu.ca

The simplest non-collision solutions of the N-body problem are the relative equilibria, in which each body follows a circular orbit around the centre of mass and the shape formed by the N bodies is constant. It is easy to see that the moment of inertia of such a solution is constant. In 1970, D. Saari conjectured that the converse is also true for the planar Newtonian N-body problem: relative equilibria are the only constant-inertia solutions. In this talk we present results related to two generalizations of Saari’s conjecture. First, we find necessary and sufficient conditions for relative equilibria of general mechanical systems with symmetry, and we relate these conditions to those for constant inertia trajectories. Second, we ask whether Saari’s conjecture, when true, is surprising. We prove that, interpreted in the context of differential topology, some appropriately generalised conjectures are generically true.

Sánchez–Morgado, Hector

Limit of the discounted Hamilton–Jacobi equation

Instituto de Matemáticas, UNAM, MX
hector@matem.unam.mx

Consider a C^3 strictly convex and superlinear Lagrangian on the d-dimensional torus $L : \mathbb{T}^d \times \mathbb{R}^d \to \mathbb{R}$ and its associated Hamiltonian $H : \mathbb{T}^d \times \mathbb{R}^d \to \mathbb{R}$,

$$H(x,p) = \max_v p.v - L(x,v)$$

We address the problem of convergence of viscosity solutions of

$$\lambda u + H(x, Du) = c$$

as λ tends to zero.

Villegas-Blas, Carlos

On the regularization of the Kepler problem and coherent states.

Instituto de Matemáticas-Cuernavaca, MX
villegas@matcuer.unam.mx

On base of work by M. Kummer, we will consider a canonical transformation relating two different ways to regularize the Kepler problem: the Moser and Kustaanheimo-Stiefel regularizations. On base of a generating function of such a canonical transformation and the moment map of the group SU(2,2), we define a suitable Bargmann transform for the Hilbert space of square integrable functions on the 3-sphere. This will be actually a coherent states transform. We will describe some semiclassical concentration properties of those coherent states.