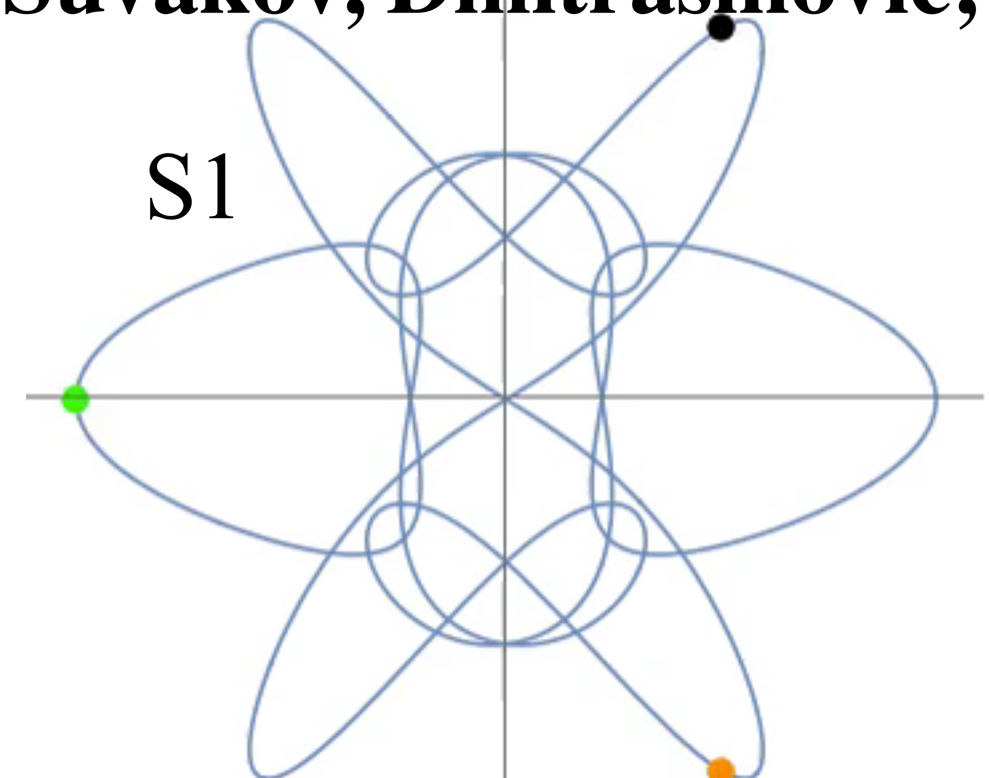
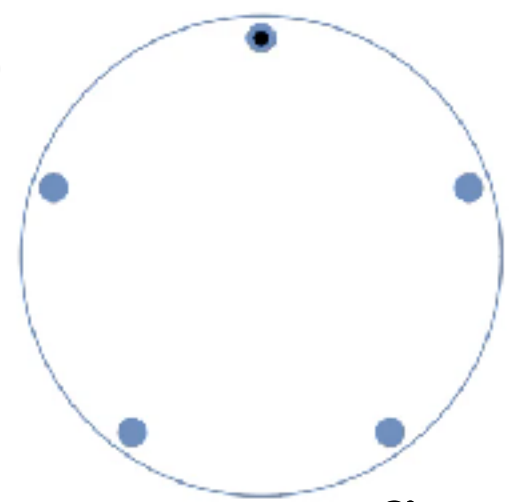
Slalom solutions and figure-eight(5)

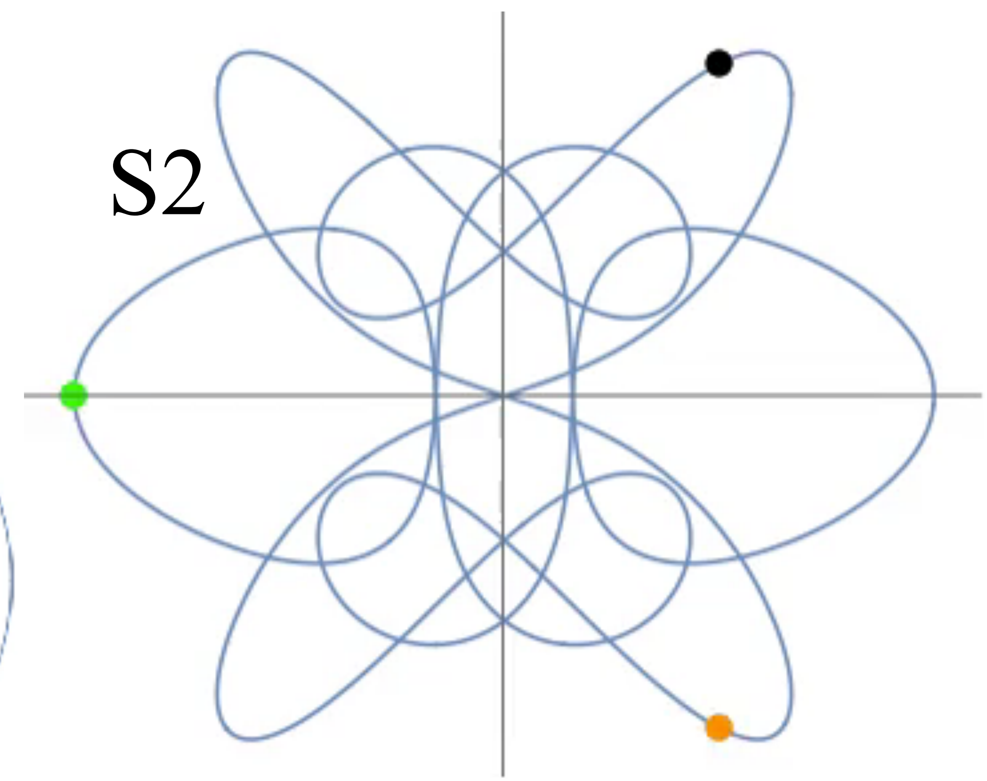
Šuvakov, Dmitrašinović, 2013

S1





S2



S3

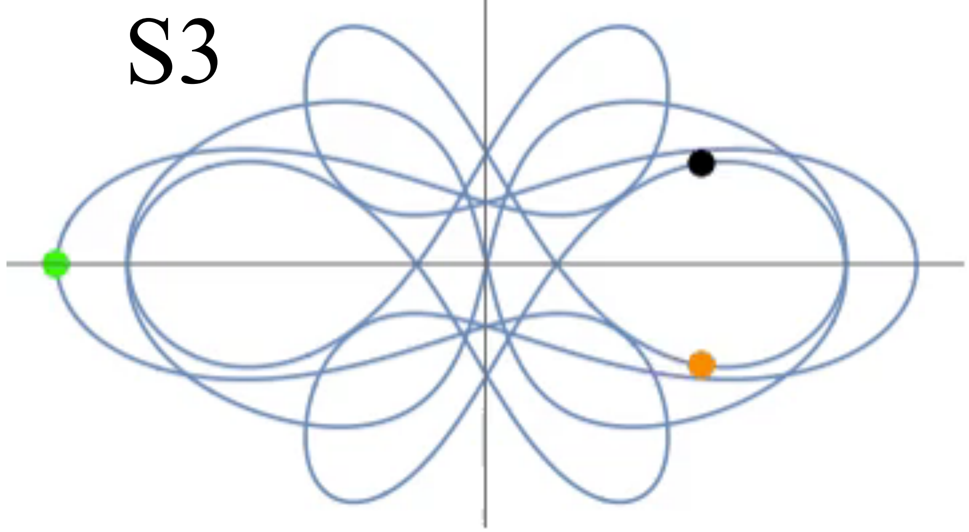
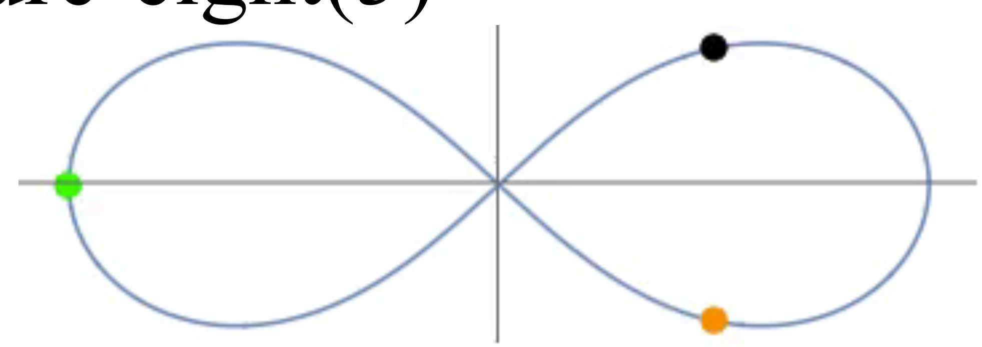


figure-eight(5)



Numerical investigations of Slalom solutions

Toshiaki Fujiwara

2017/03/22

at Tokyo Institute of Technology

Slalom Solutions

- **Šuvakov, M.**

  Numerical search for periodic solutions in the vicinity of the figure-eight orbit: slaloming around singularities on the shape sphere, **Celest. Mech. Dyn. Astron. 119, 369–377 (2014)**

- **Šuvakov, M., Dmitrašinović, V.**

  Three classes of Newtonian three-body planar periodic orbits**, Phys. Rev. Lett. 110(11), 114301 (2013)**

- **Šuvakov, M., Dmitrašinović, V.**

  A guide to hunting periodic three-body orbits**, Am. J. Phys. 82, 609–619 (2014)**

Three-body choreography

$$L = \frac{1}{2}\sum_k \left|\frac{dq_k}{dt}\right|^2 + \frac{1}{\alpha}\sum_{i,j}\frac{1}{|q_i - q_j|^\alpha}$$

$\alpha = 1:$ Newton potential

$$q_0(t) = q(t), q_1(t) = q(t + T/3), q_2(t) = q(t + 2T/3)$$

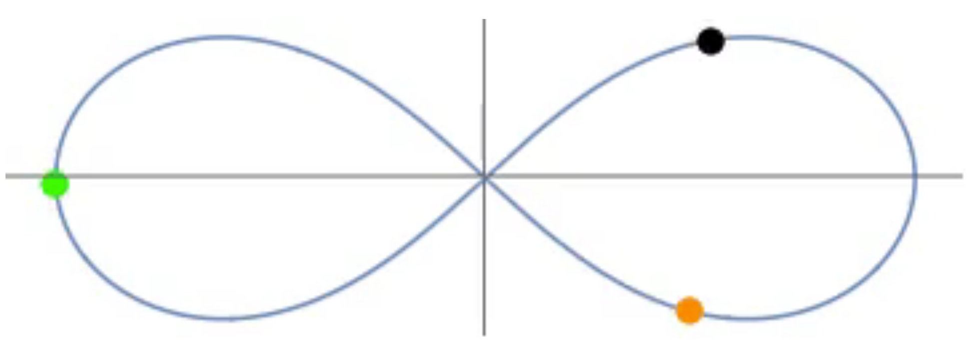
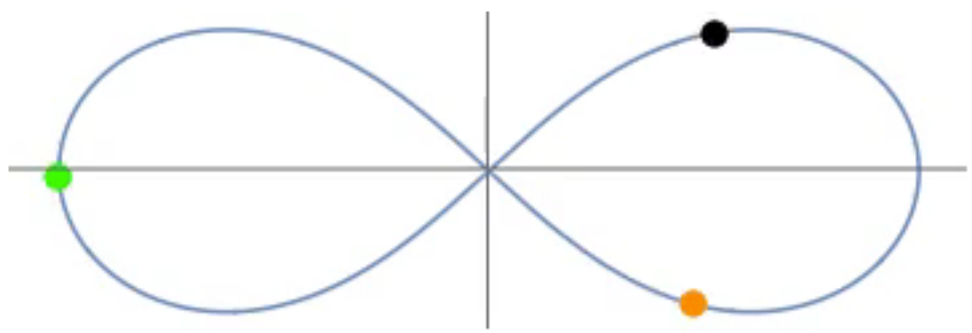


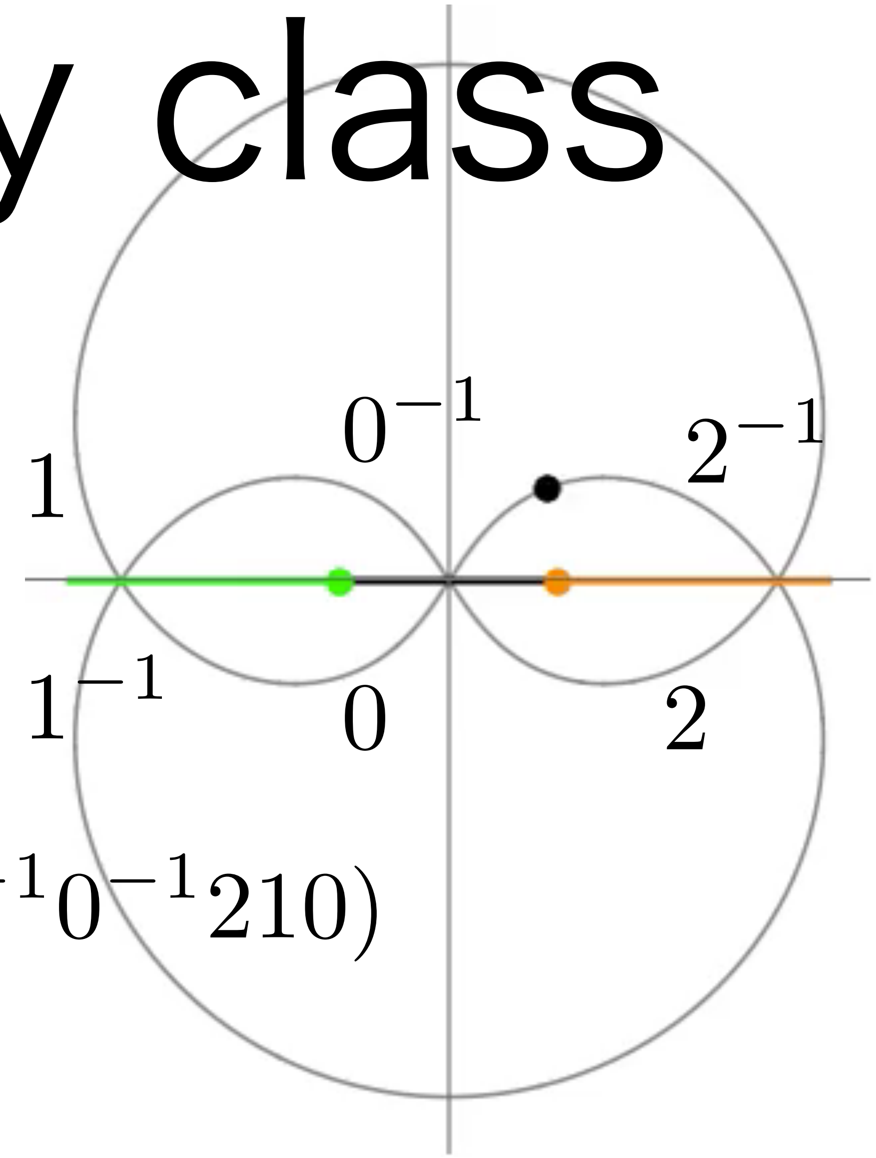
figure-eight solution

C. Moore 1993,

A. Chenciner and R. Montgomery 2000

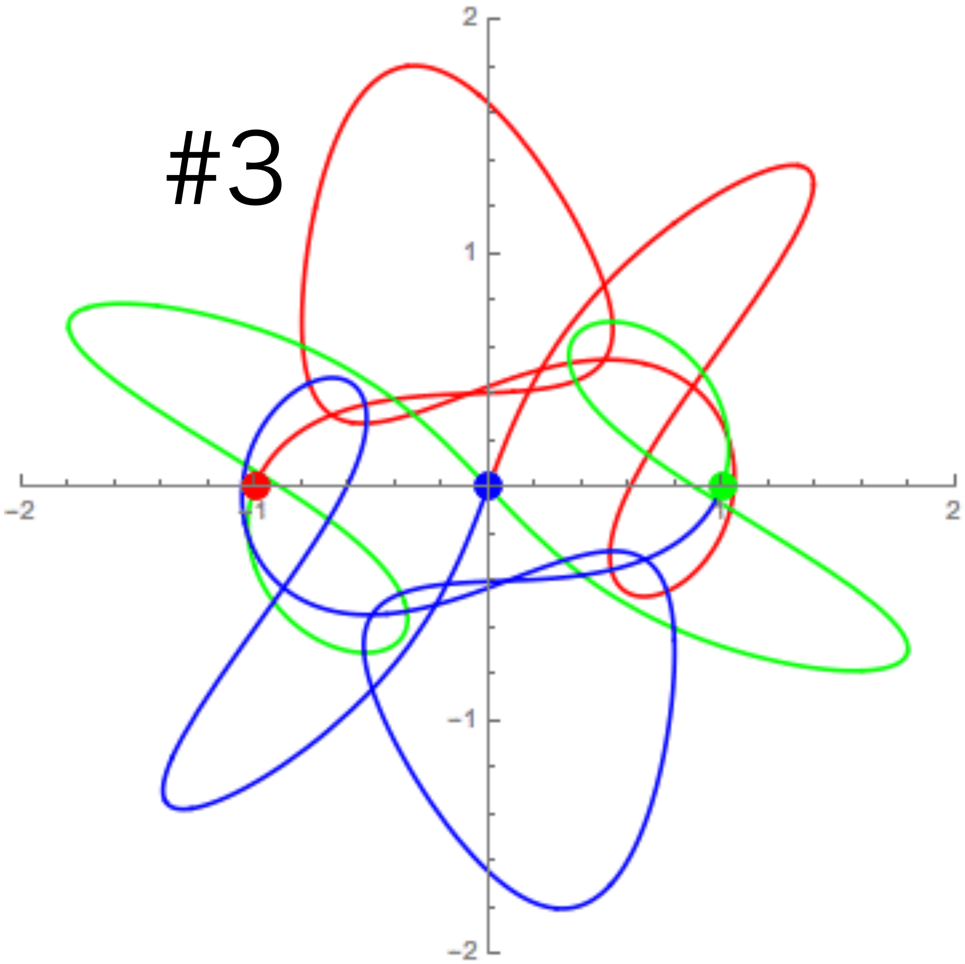
homotopy class

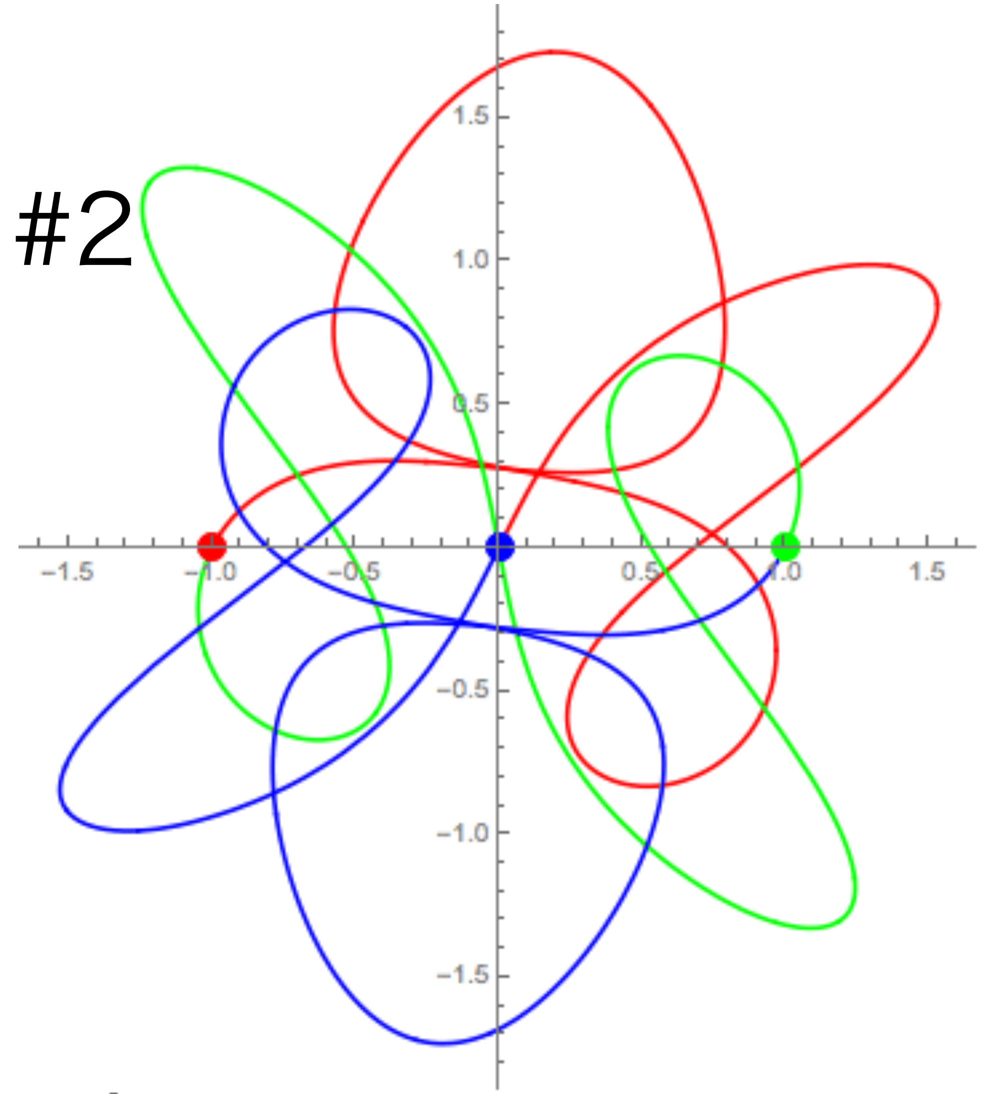


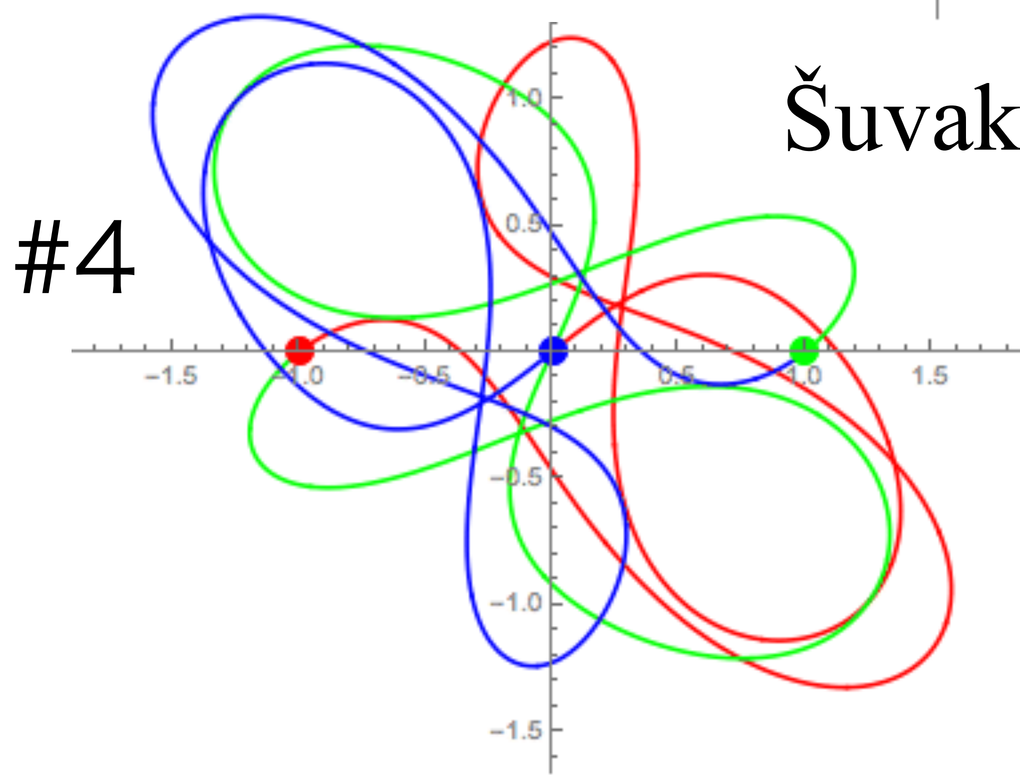


$(2^{-1}1^{-1}0^{-1}210)$

k-slalom: $(2^{-1}1^{-1}0^{-1}210)^k$







Šuvakov, Dmitrašinović, 2013

5-slalom

$(2^{-1}1^{-1}0^{-1}210)^5$

T/T_fig8 for k=5

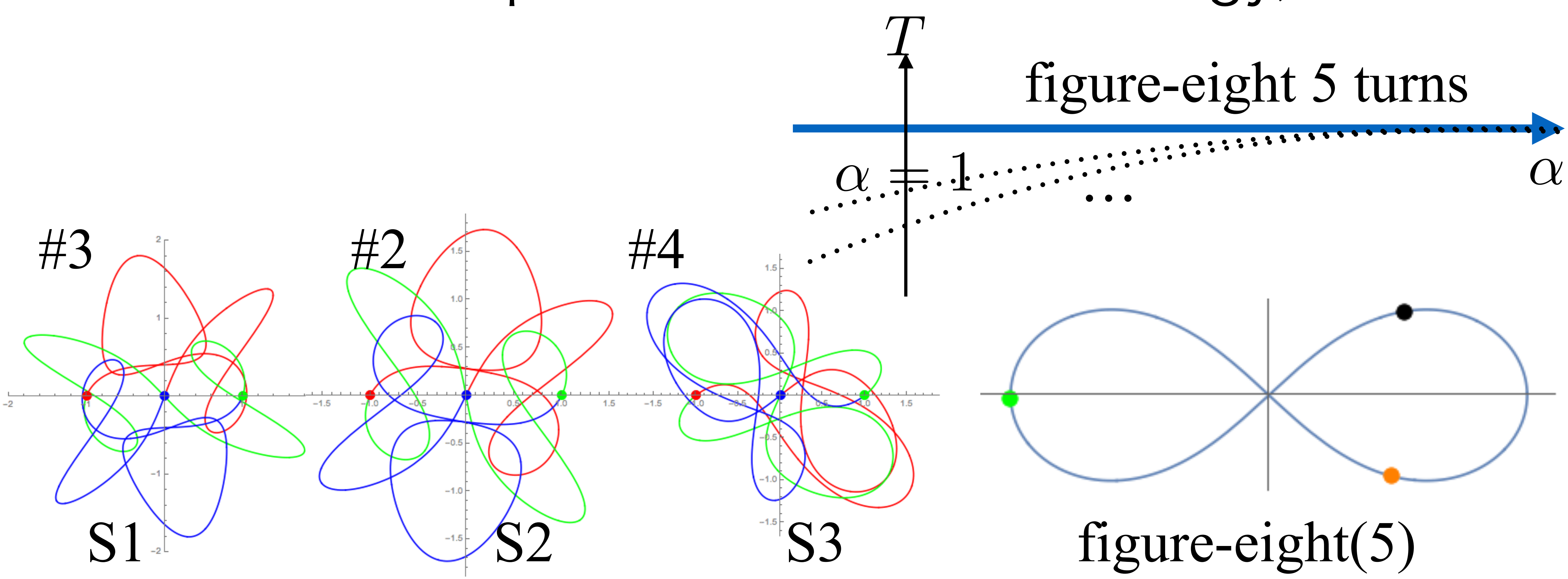
Šuvakov-Dmitrašinović, Šuvakov-Shibayama

name	T	T/T_fig8	Šuvakov-Shibayama
	26.1281	1	#1=figure8
S1	130.146	4.98106	#3
S2	130.149	4.98118	#2
S3	130.288	4.98652	#4

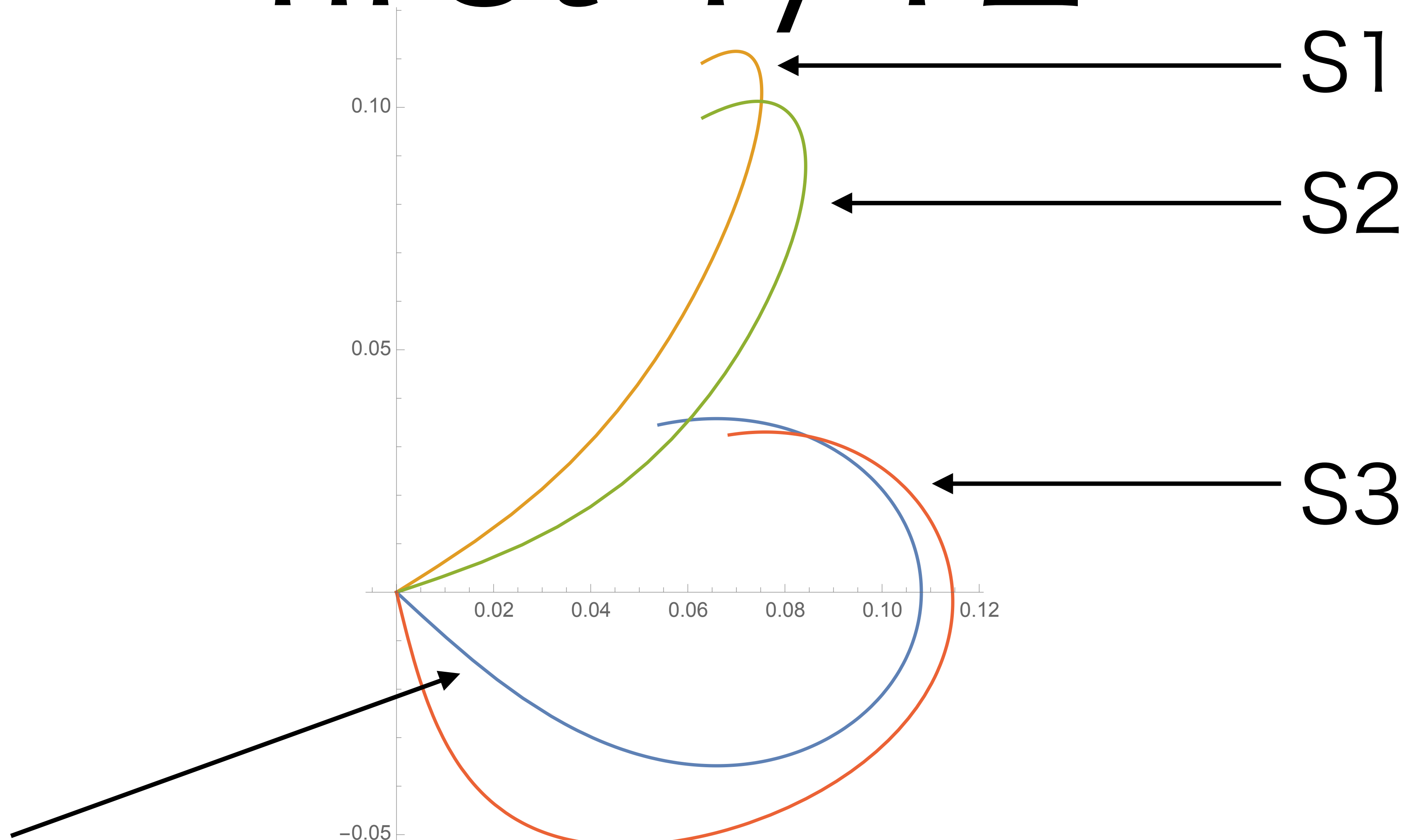
T is for $E = -1/2$ → $T/T_{\mathrm{fig8}} \sim k$

my first speculation

slalom solutions with k=5 and
figure-eight solution 5 turns
have the same homotopy class and
similar period for common energy, so ⋯



first T/12



first $T/12$ for figure-eight(5)= first $5T_{\mathrm{fig8}}/12$ for figure-eight

I guess …

$$k = 5 + 6n = \mathbf{5}, \mathbf{11}, \mathbf{17}, 23, \ldots$$
$$k = 1 + 6n = \mathbf{7}, 13, 19, 25, \ldots$$

while, I don't know how $k = \mathbf{22}, \mathbf{26}, \mathbf{35}, \mathbf{41}$

Precise value of ratio of T for **common energy**

First, we determination the orbit for $T=1$ with $|q_k(0)-q_k(T)|<10^{-50}$

easier for numerical calculations

Then,

method 1

scaling $E \to E=-1/2$, then $\dfrac{T}{T_{\text{fig8}}}$

scale invariant

method 2

directly calcurate $J=\int_0^T \sum |\dot{q}_k|^2 dt = \dfrac{D}{(-E)^{(2-\alpha)/(2\alpha)}}$

$$T=\frac{dJ}{dE}=\left(\frac{2-\alpha}{2\alpha}\right)\frac{D}{(-E)^{(2+\alpha)/(2\alpha)}} \quad \Rightarrow \frac{T}{T_{\text{fig8}}}=\frac{D}{D_{\text{fig8}}}$$

$T\&E$, $J\&E$ for any scale

Precise value of ratio of T for **common energy**

S1	T/Tfig8	4.98118
	T/Tfig8	4.98130390178020770375
	D/Dfig8	4.98130390178020770375
S2	T/Tfig8	4.98106
	T/Tfig8	4.98096119200589014111
	D/Dfig8	4.98096119200589014111
S3	T/Tfig8	4.98652
	T/Tfig8	4.98651794945926031160
	D/Dfig8	4.98651794945926031160

Trace the solutions

$$L = \frac{1}{2}\sum_k \left|\frac{dq_k}{dt}\right|^2 + \frac{1}{\alpha}\sum_{i,j}\frac{1}{|q_i - q_j|^\alpha}$$

change α with $\Delta\alpha = 10^{-3}$ or 10^{-4}

α: initial condition for α

||

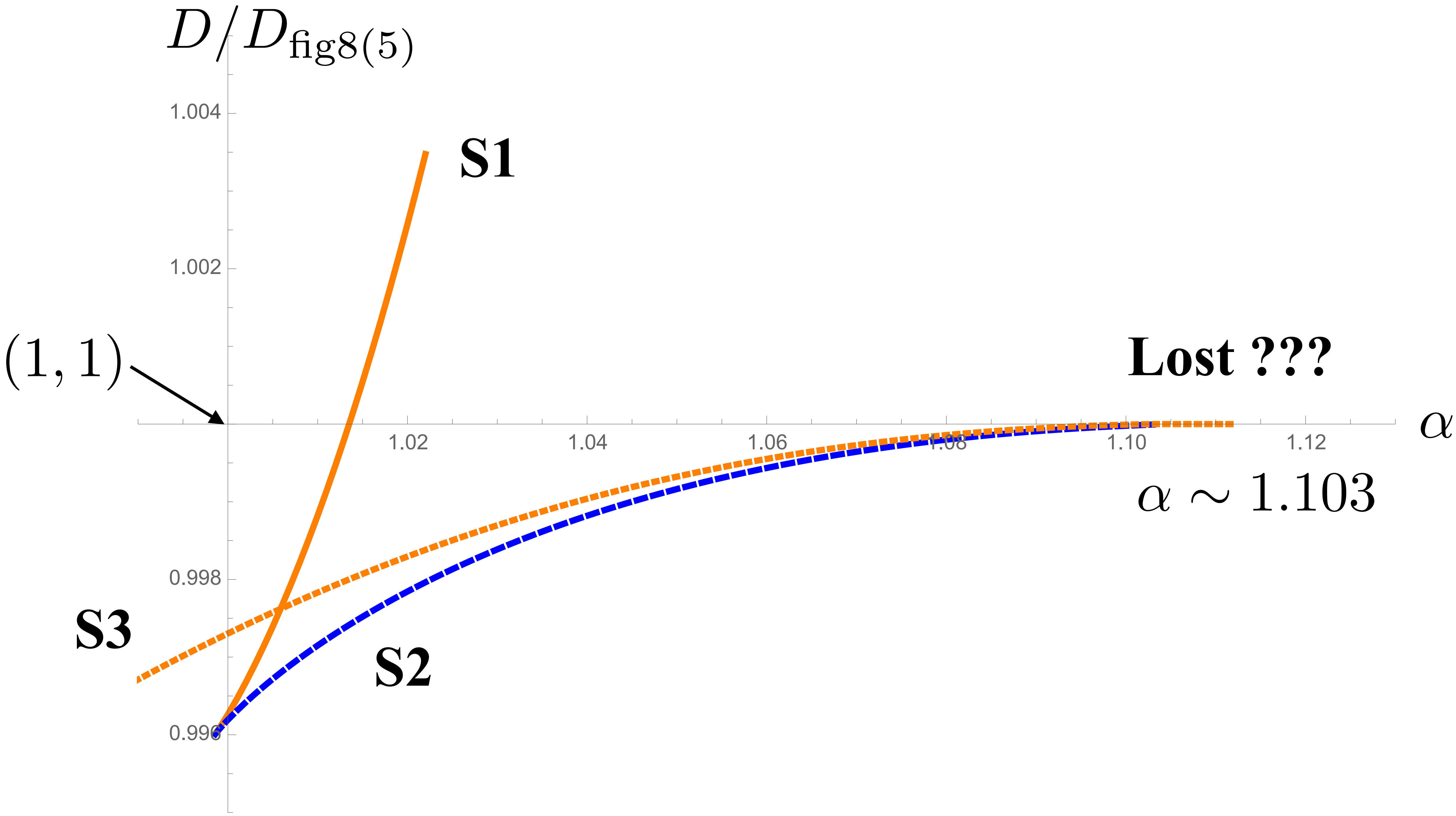
$\alpha + \Delta\alpha$: start from initial condition for α

improve by Newton method

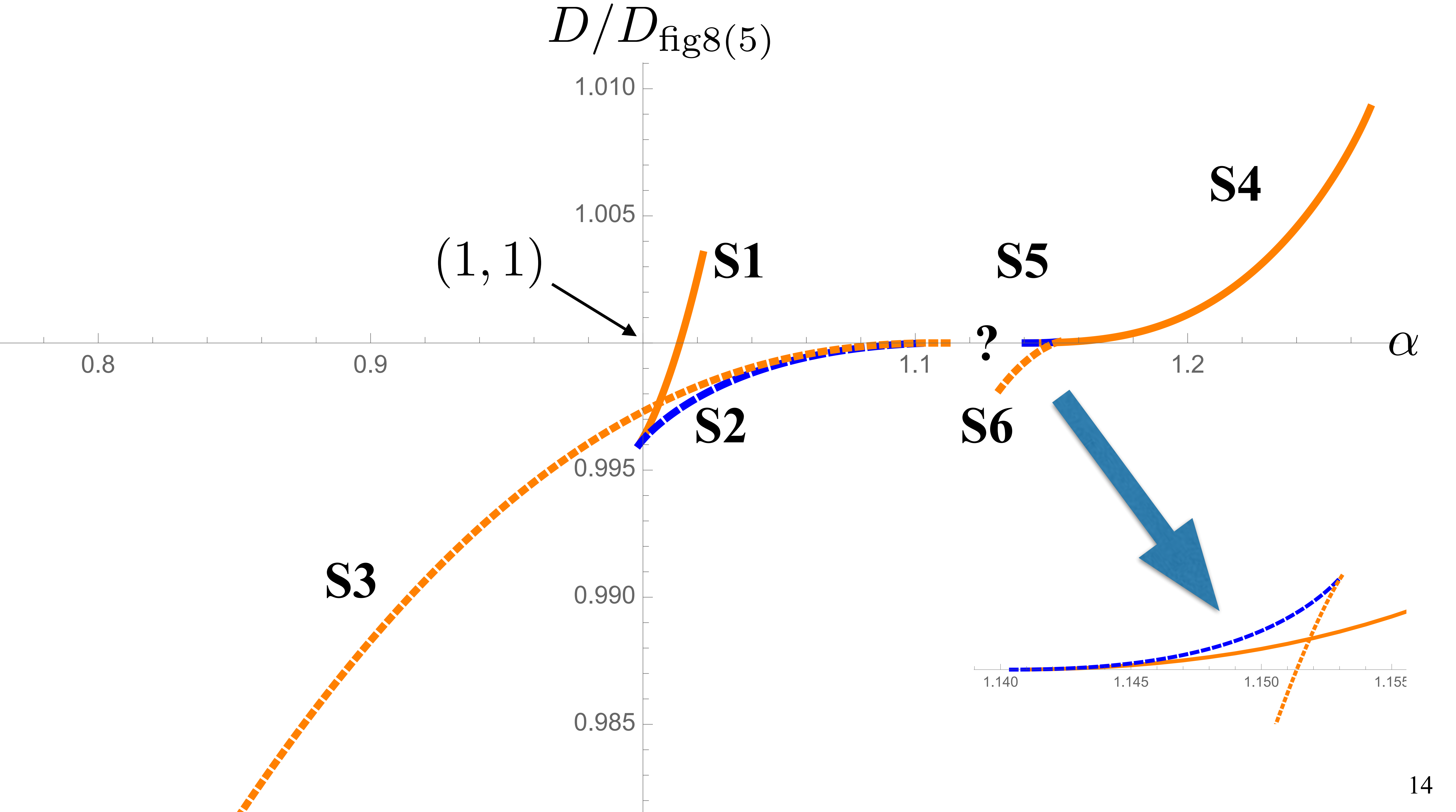
initial condition for $\alpha + \Delta\alpha$

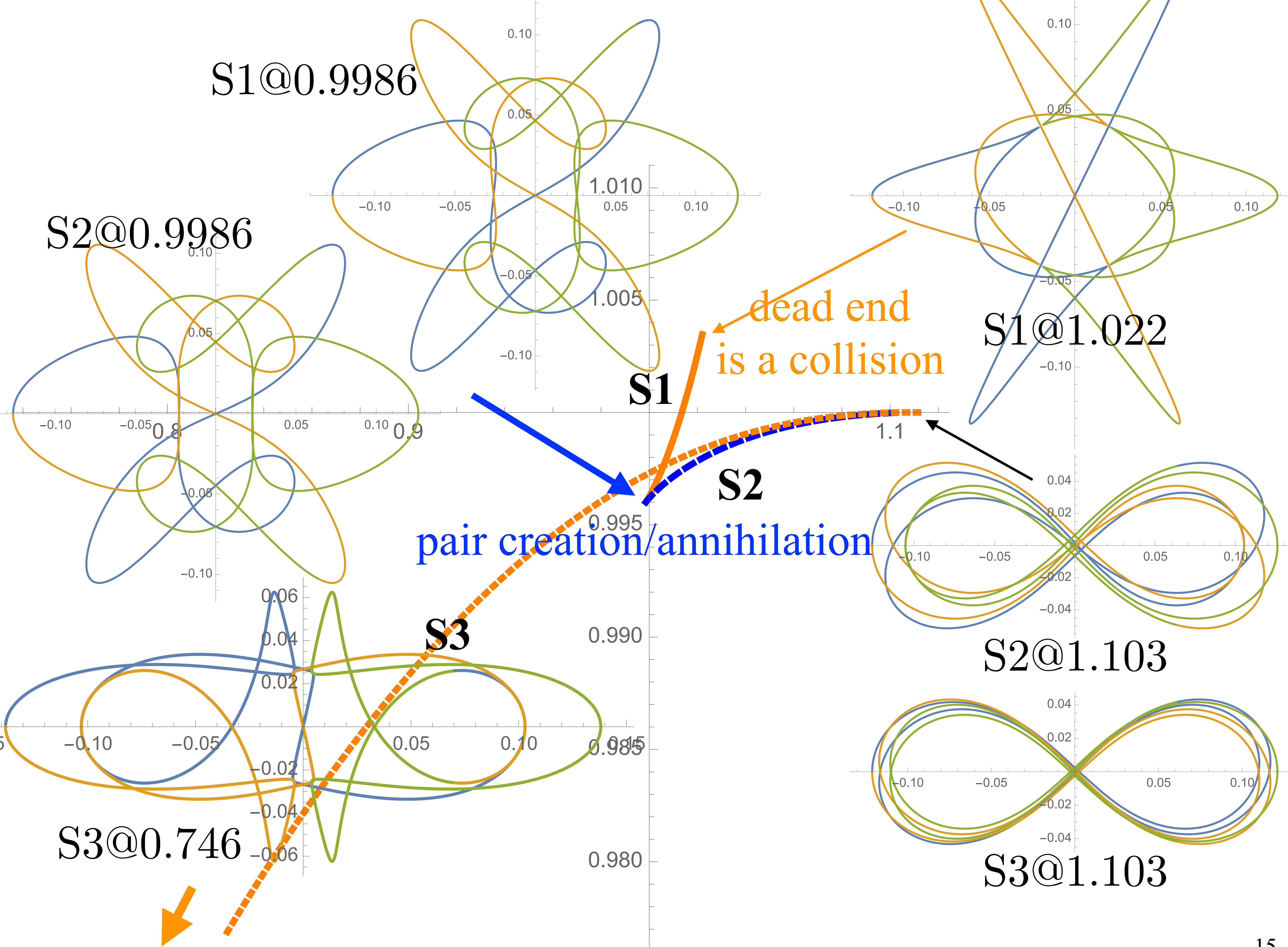
$\alpha \leftarrow \alpha + \Delta\alpha$

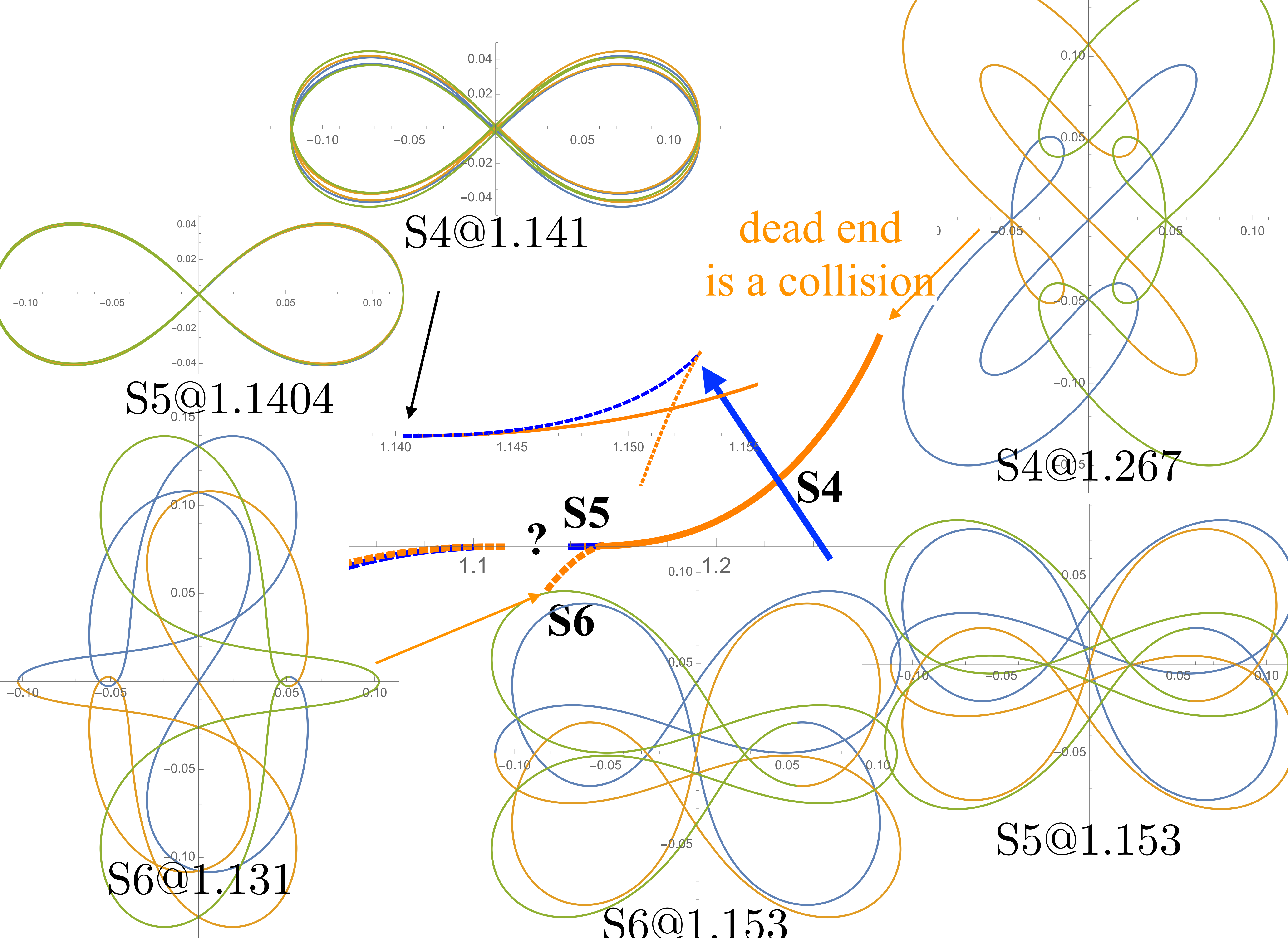
Trace the solutions



Trace the solutions

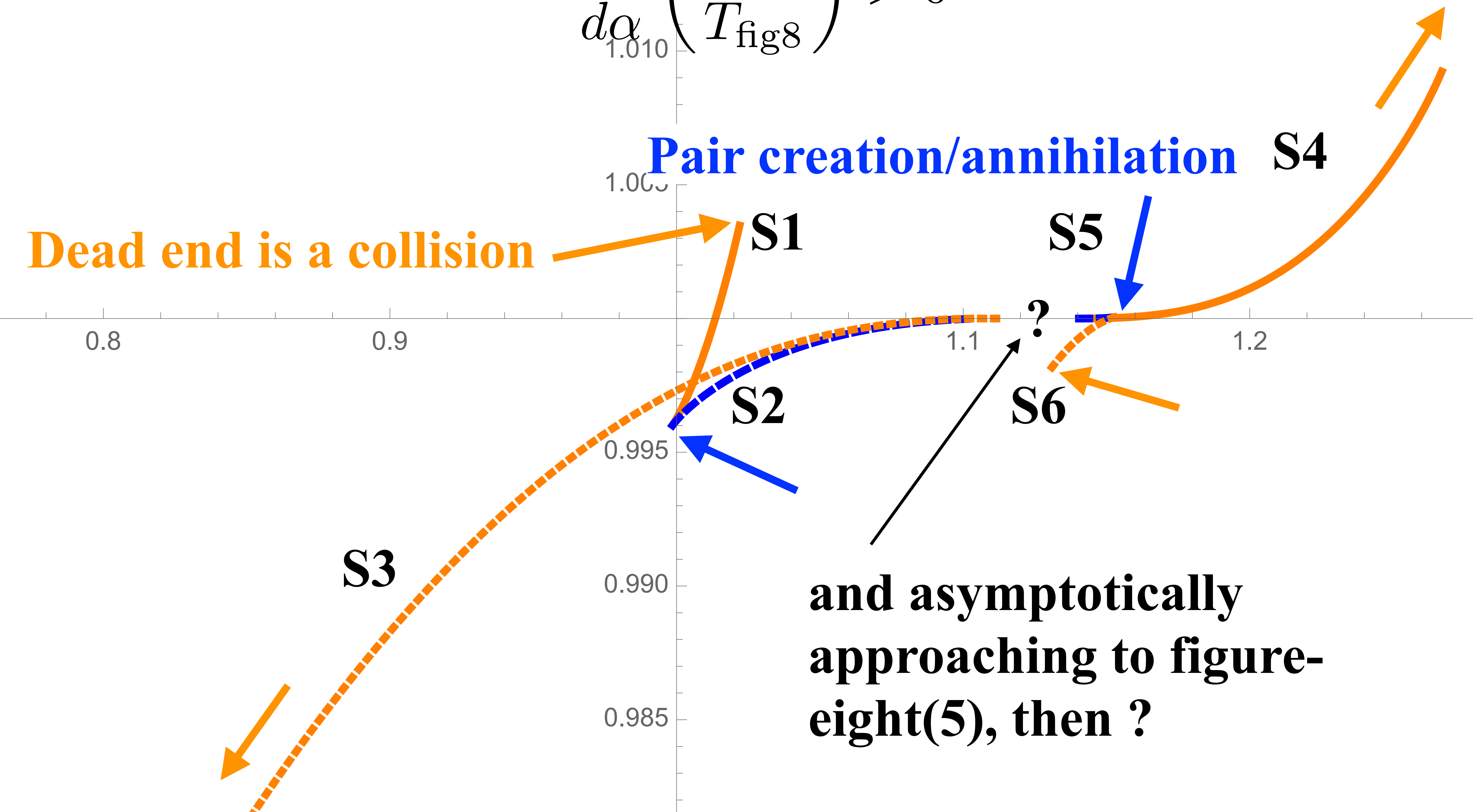



S1@0.9986
S2@0.9986
S1@1.022
dead end
is a collision
S1
S2
S3
pair creation/annihilation
S2@1.103
S3@1.103
S3@0.746

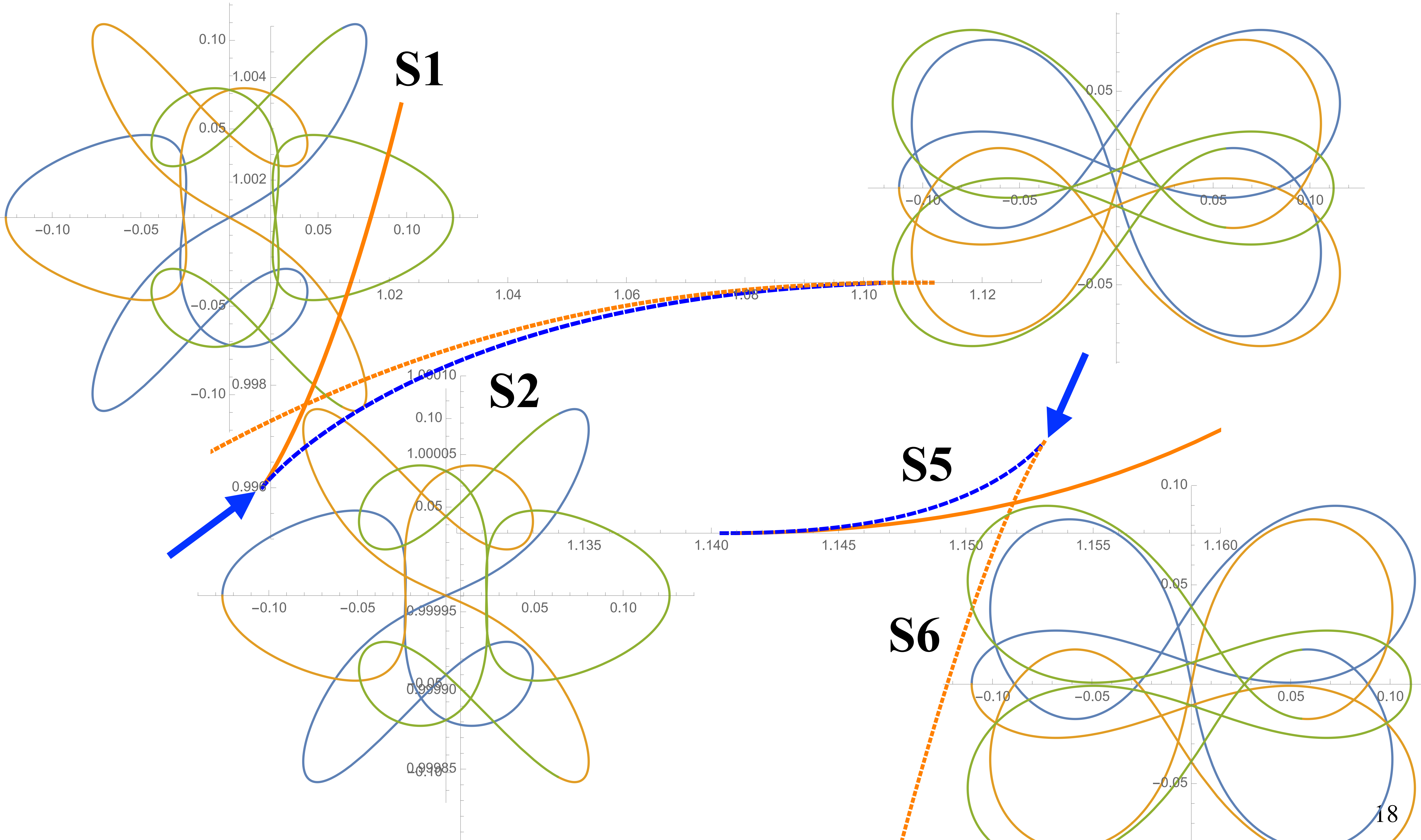

S4@1.141
S5@1.1404
dead end
is a collision
S4@1.267
S5
S4
?
S6
S6@1.131
S6@1.153
S5@1.153
1.140
1.145
1.150
1.1
1.2

Trace the solutions

$$\frac{d}{d\alpha}\left(\frac{T}{T_{\rm fig8}}\right) > 0$$

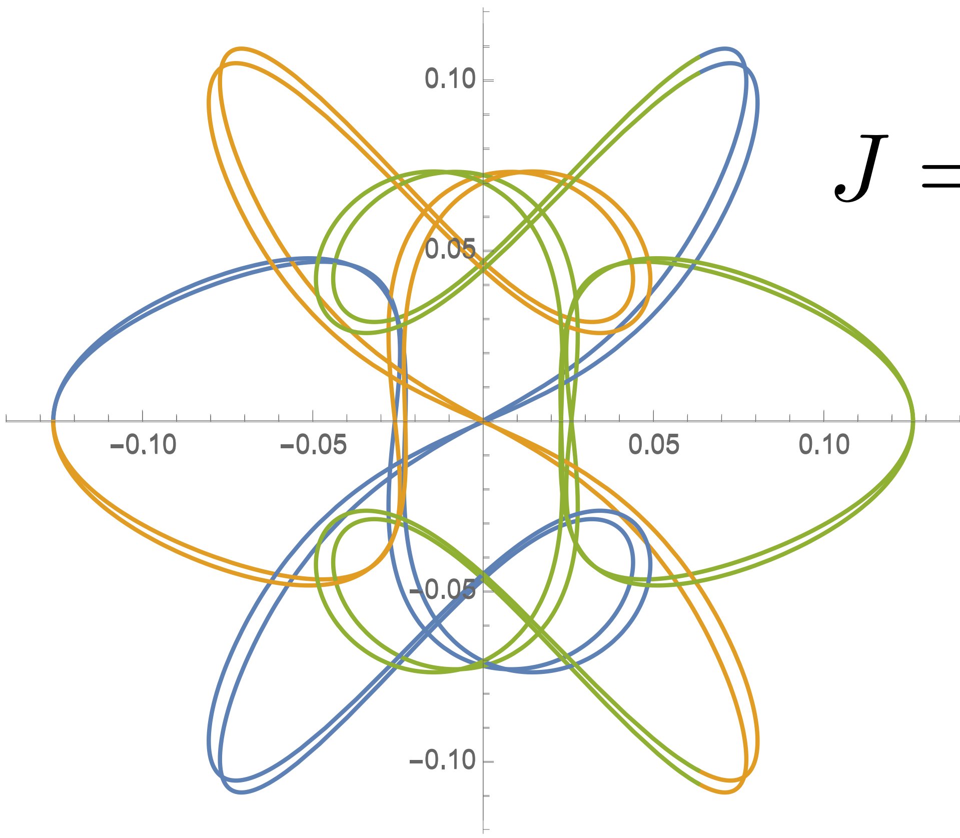


Pair creation/annihilation



S1 and S2

$\alpha = 0.9986$



$$J = \int_0^T \sum |\dot{q}_k|^2 dt = \frac{D}{(-E)^{(2-\alpha)/(2\alpha)}}$$

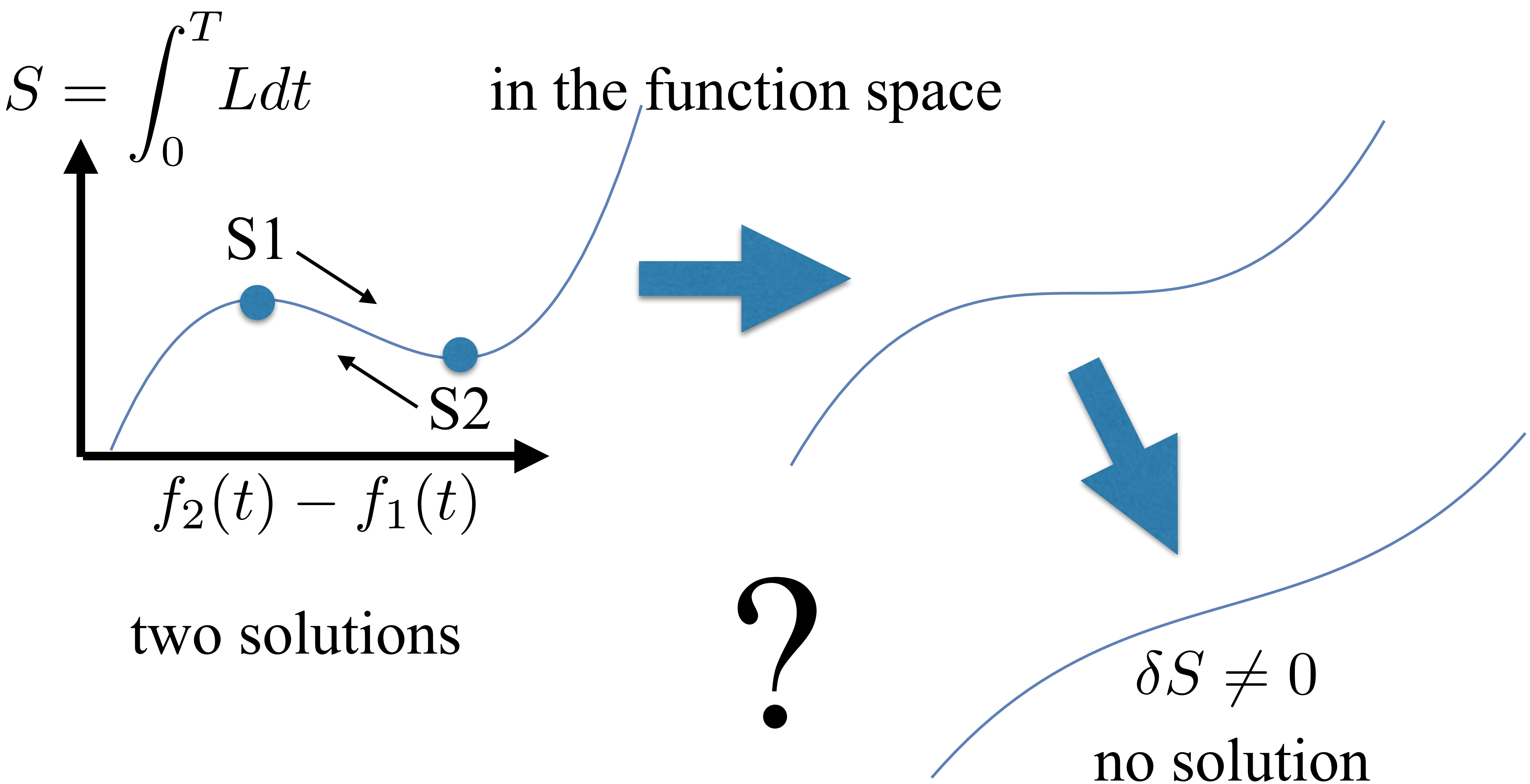
$J_1 = 25.62492035$
$J_2 = 25.62491358$ for $T = 1$

$D_1 = 92.18098947$
$D_2 = 92.18095288$

pair creation/annihilation at $\alpha < 0.9986$

why/how ?

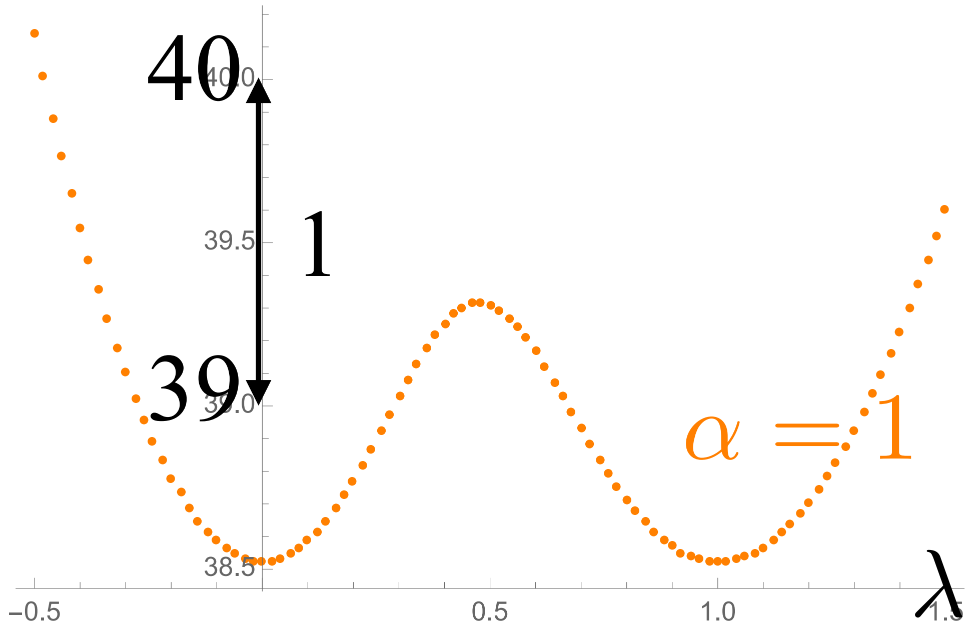
a scenario/speculation

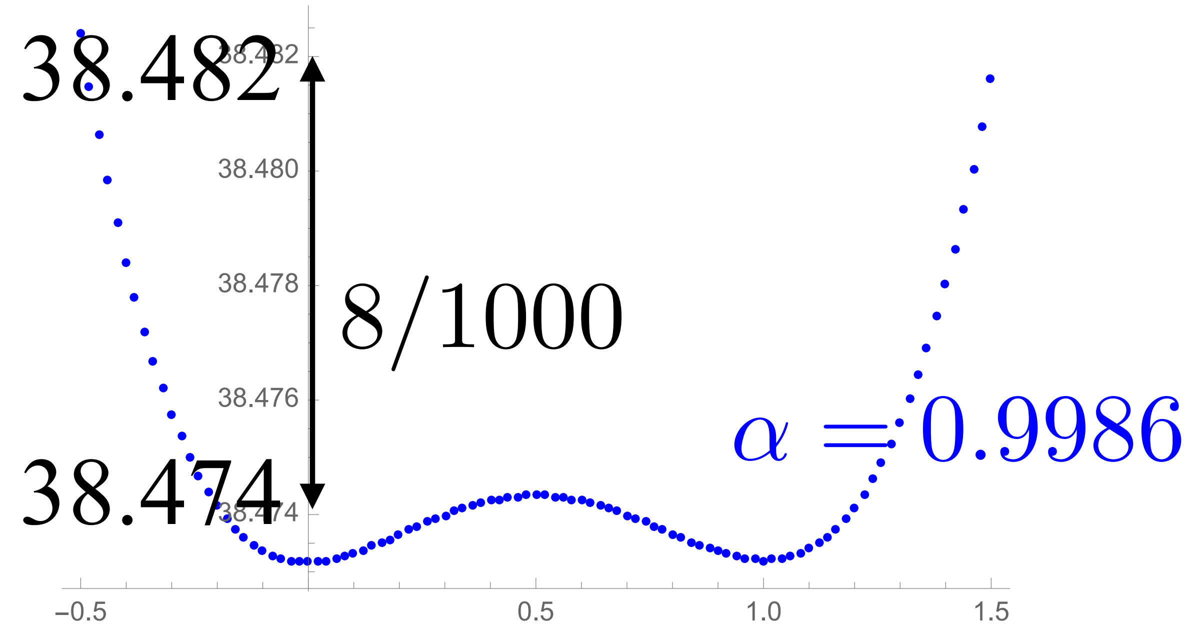


calculated action

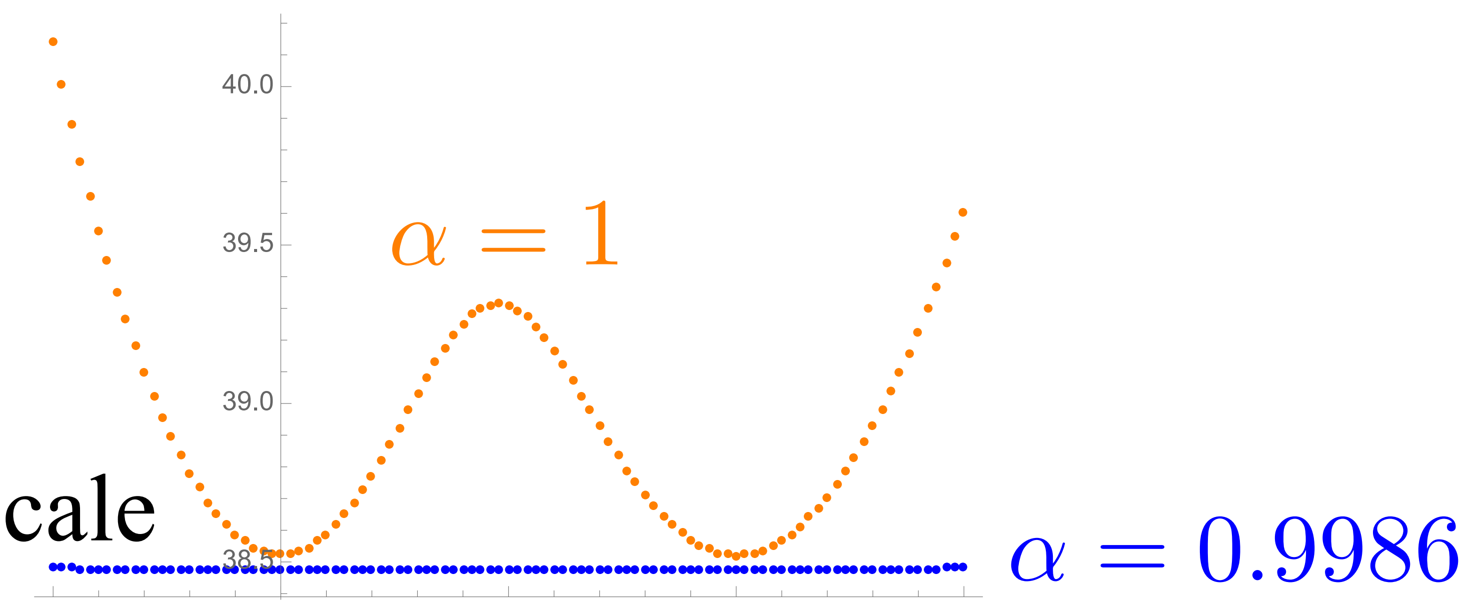
in the function space

$S[f_1 + \lambda(f_2 - f_1)]$





in a common scale



eigenvalue of Hessian of Lagrangian

second order variation of action

$$S[q+\delta q] = S[q] + \frac{1}{2}\int_0^T dt\, \delta q H \delta q + ..., \; H = -\frac{d^2}{dt^2} + \frac{\partial^2 U}{\partial q \partial q}$$

eigenvalue and eigenfunction of $H\Psi = \lambda \Psi$

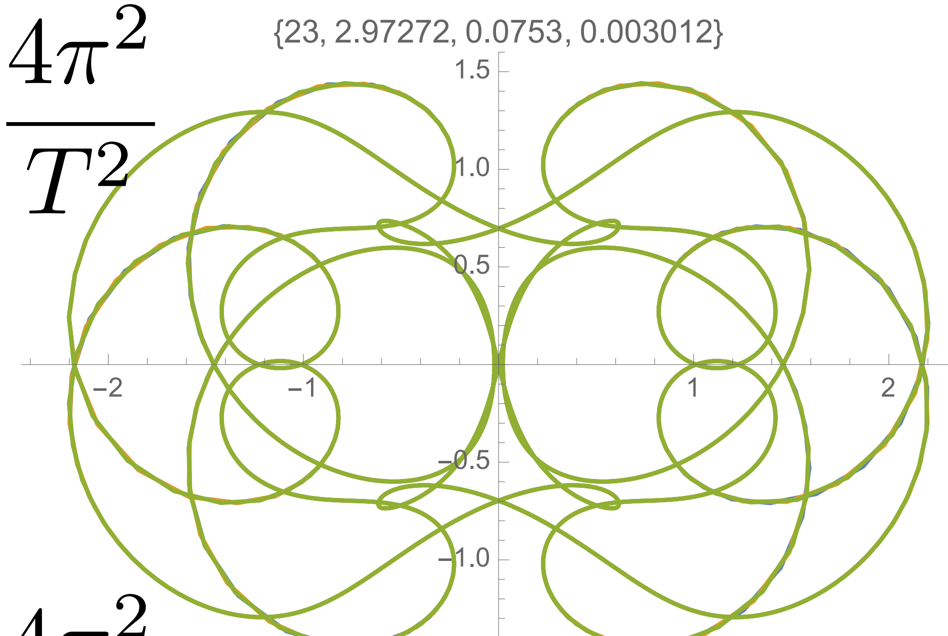
H: real symmetric 6×6 matrix,

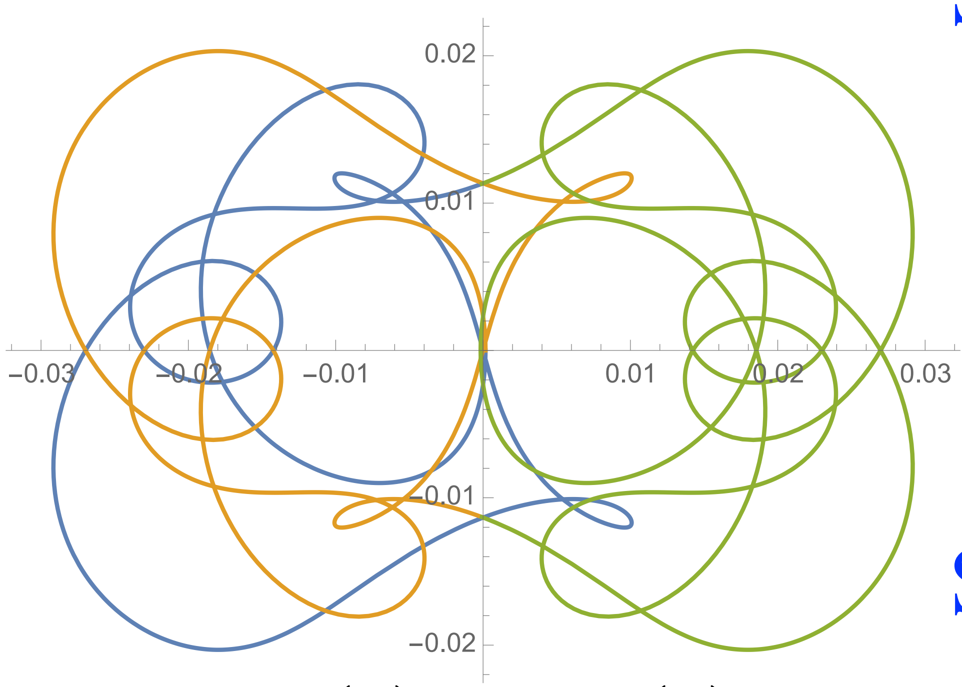
$$\Psi = \begin{pmatrix} \delta q_0 \\ \delta q_1 \\ \delta q_2 \end{pmatrix}, \; \delta q_k = \begin{pmatrix} \delta x_k \\ \delta y_k \end{pmatrix}$$

preliminary eigenvalue/eigenfunction

$\alpha = 0.9986$ $\quad H\Psi = \lambda\Psi$

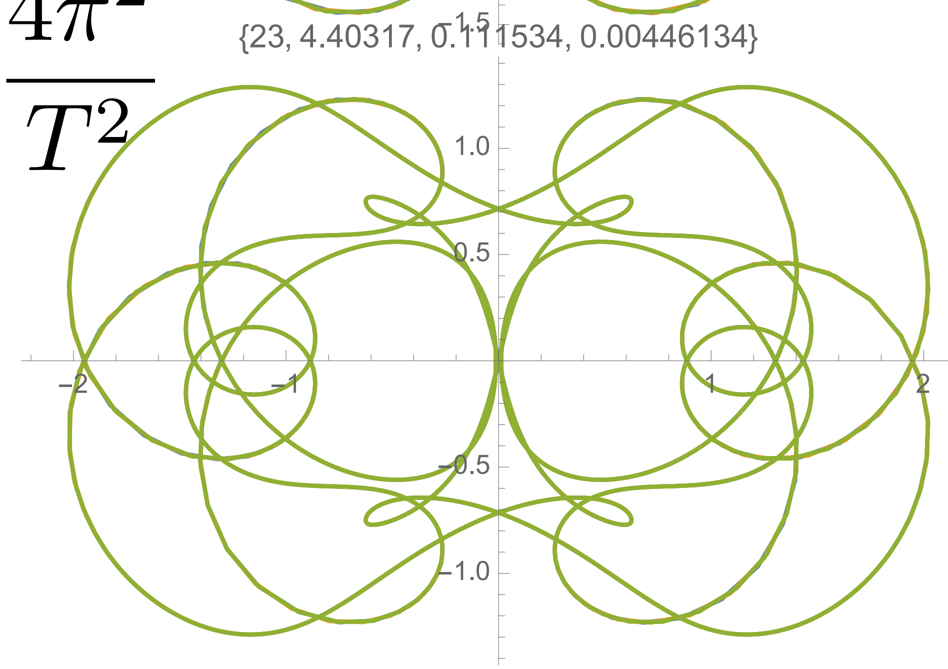
S1: $\lambda_{23} = 0.0753 \times \dfrac{4\pi^2}{T^2}$





$f_2(t) - f_1(t)$

S2: $\lambda_{23} = 0.1115 \times \dfrac{4\pi^2}{T^2}$



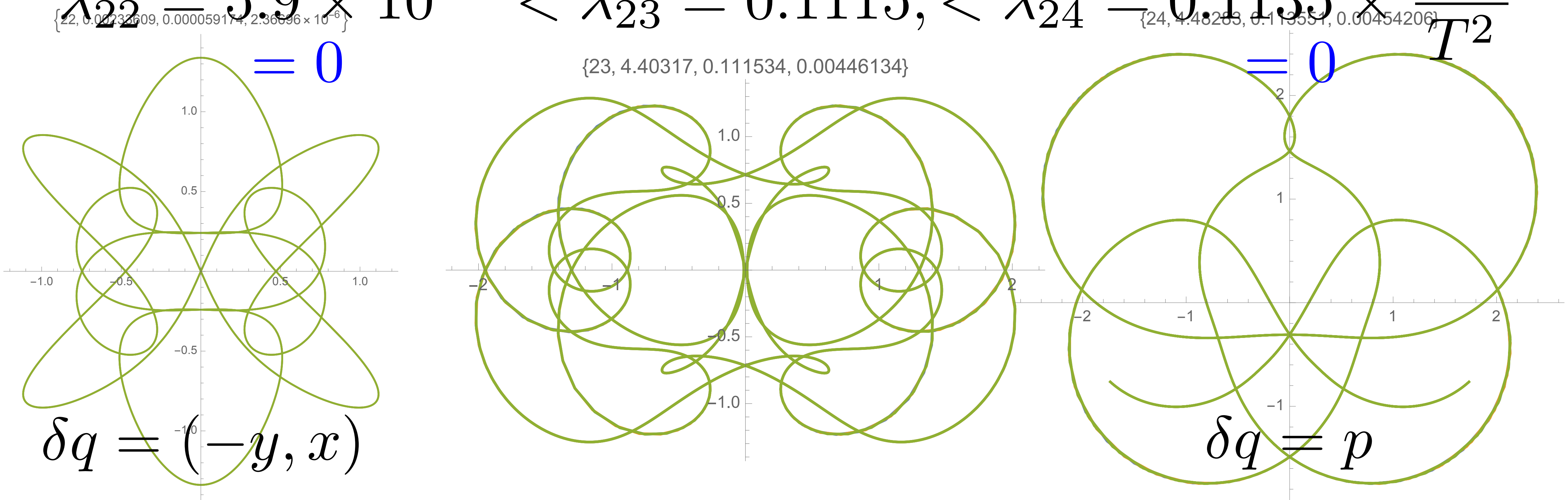
λ_{23} are not inconsistent with zero
~~consistent~~

accuracy of eigenvalues

S2: $\alpha = 0.9986$ $H\Psi = \lambda\Psi$

$$\lambda_{22} = 5.9 \times 10^{-6} < \lambda_{23} = 0.1115, < \lambda_{24} = 0.1135 \times \frac{4\pi^2}{T^2}$$

$= 0$ $\qquad$ $= 0$



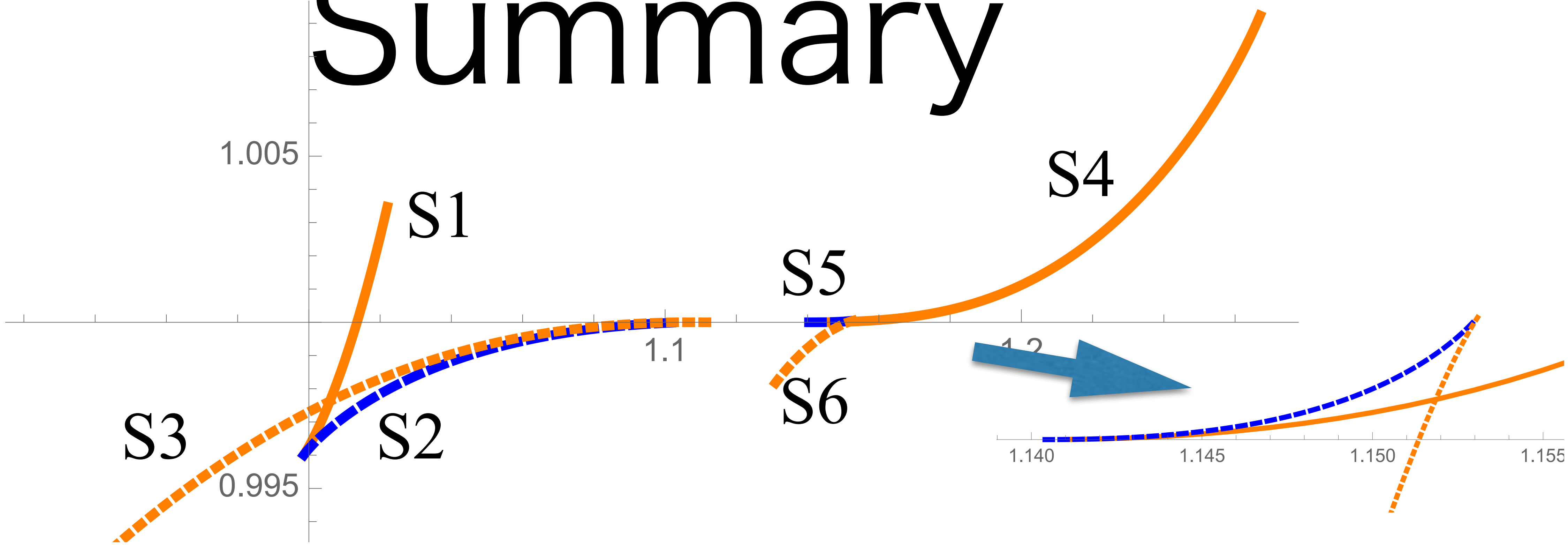
$\delta q = (-y, x)$ $\qquad$ $\delta q = p$

λ_{23} are not inconsistent with zero

we need more and more accurate value

→ spectroscopy of Hessian

Summary



- We trace the 5-slalom solutions S1, S2, S3 and S4, S5, S6 for alpha ~ 1
- T/Tfig8 for common E are not constant $\frac{d}{d\alpha}\left(\frac{T}{T_{\mathrm{fig8}}}\right) > 0$
- Dead end of S1, S3, S4, S6 are collision
- T/Tfig8 for S2, S3 and S4, S5 asymptotic approach to 5, **then ?**
- **Pair annihilation/creation of S1, S2 and S5, S6**

… we are trying to find a mechanism in the function space