Choreograpgy on the Lemniscate

$$q(t) = \left( \frac{\mathrm{sn}(t)}{1+\mathrm{cn}^2(t)}, \frac{\mathrm{sn}(t)\mathrm{cn}(t)}{1+\mathrm{cn}^2(t)} \right) \text{ with } k^2 = \frac{2+\sqrt{3}}{4},$$

$$\begin{cases} q_1(t) = q(t), \\ q_2(t) = q(t+T/3), \\ q_3(t) = q(t+2T/3), \end{cases}$$

satisfies the equation of motion $\ddot{q}_i = -\dfrac{\partial}{\partial q_i} U$ with

$$U = \sum_{i<j} \left( \frac{1}{2} \ln r_{ij} - \frac{\sqrt{3}}{24} r_{ij}^2 \right).$$