

# The list of Lagrange shapes with $(\sigma_1, \sigma_2, \sigma_3) = (i, j, k)\pi/12, 1 \leq i \leq j \leq k < 12$

Toshiaki Fujiwara, Ernesto Pérez-Chavela

November 8, 2023

**Definition 1** (Shape). *For three body problem on  $S^2$ , a shape is the set of arc angles  $\{\sigma_1, \sigma_2, \sigma_3\}$ , where  $\sigma_k$  is the angle between the mass  $m_i$  and  $m_j$  seen from the origin of  $S^2$ , for  $(i, j, k) = (1, 2, 3), (2, 3, 1)$ , and  $(3, 1, 2)$ .*

**Definition 2** (Lagrange shape and Euler shape). *A shape is called Langange (Euler) shape if and only if the shape can form Lagrange (Euler) relative equilibrium.*

In the following, the Lagrange shapes with  $(\sigma_1, \sigma_2, \sigma_3) = (i, j, k)\pi/12, 1 \leq i \leq j \leq k < 12$  are listed.

To form a triangle,  $(i, j, k)$  must satisfy  $i + j > k$  and  $(i + j + k)/12 < 2$ . There are 108 triangles in this grid points.

There are 73 Lagrange shapes. The remaining 35 triangles are not Lagrange shape. Namely, the 35 triangles cannot form Lagrange relative equilibrium.

For detail, see section 5.3 in “Continuations and bifurcations of relative equilibria for the positive curved three body problem”.

## 1 List of 73 Lagrange Shapes

The format is

No.  $\{i, j, k\}, \{m_1/m_3, m_2/m_3\}, \omega^2/m_3$ .

1.  $\{1, 1, 1\}, \{1, 1\}, 12\sqrt{2}(3\sqrt{3} + 5)$
2.  $\{1, 6, 6\}, \left\{ \frac{1}{-5\sqrt{2}+4\sqrt{3}-3\sqrt{6}+8}, 1 \right\}, 2\sqrt{2}(\sqrt{3} + 1)^3 \left( \frac{(\sqrt{3}-1)^6}{512(-5\sqrt{2}+4\sqrt{3}-3\sqrt{6}+8)} + 2 \right)$
3.  $\{1, 7, 7\}, \left\{ \frac{-43\sqrt{2}+32\sqrt{3}-19\sqrt{6}+56}{67\sqrt{2}-224\sqrt{3}-37\sqrt{6}+384}, 1 \right\}, \frac{8(-1339\sqrt{2}-76\sqrt{3}+756\sqrt{6}+172)}{-192\sqrt{2}+37\sqrt{3}+112\sqrt{6}-67}$

4.  $\{1, 8, 8\}, \left\{ \frac{-3825\sqrt{2}+3240\sqrt{3}-2211\sqrt{6}+5616}{107\sqrt{2}-84\sqrt{3}+59\sqrt{6}-140}, 1 \right\}, -\frac{16(6462\sqrt{2}-5474\sqrt{3}+3726\sqrt{6}-9465)}{9(107\sqrt{2}-84\sqrt{3}+59\sqrt{6}-140)}$
5.  $\{1, 9, 9\}, \left\{ \frac{-832\sqrt{2}+654\sqrt{3}-480\sqrt{6}+1134}{26\sqrt{2}-23\sqrt{3}+16\sqrt{6}-39}, 1 \right\}, \frac{4(1056\sqrt{2}-895\sqrt{3}+608\sqrt{6}-1554)}{-39\sqrt{2}+32\sqrt{3}-23\sqrt{6}+52}$
6.  $\{1, 10, 10\}, \left\{ \frac{-245\sqrt{2}+208\sqrt{3}-141\sqrt{6}+360}{17\sqrt{2}-12\sqrt{3}+11\sqrt{6}-28}, 1 \right\}, -\frac{16(106\sqrt{2}-91\sqrt{3}+62\sqrt{6}-162)}{17\sqrt{2}-12\sqrt{3}+11\sqrt{6}-28}$
7.  $\{1, 11, 11\}, \{10\sqrt{2} - 8\sqrt{3} + 6\sqrt{6} - 13, 1\}, -508\sqrt{2}+480\sqrt{3}-292\sqrt{6}+832$
8.  $\{2, 2, 2\}, \{1, 1\}, 24$
9.  $\{2, 6, 6\}, \left\{ \frac{1}{2}(\sqrt{3} + 2), 1 \right\}, \frac{1}{16}(\sqrt{3} + 258)$
10.  $\{2, 7, 7\}, \left\{ \frac{1}{11}(17\sqrt{3} + 30), 1 \right\}, \frac{1}{11}(-8)(8\sqrt{3} - 37)$
11.  $\{2, 8, 8\}, \left\{ \frac{3}{107}(124\sqrt{3} + 15), 1 \right\}, \frac{8}{963}(124\sqrt{3} + 1941)$
12.  $\{2, 9, 9\}, \left\{ \frac{2\sqrt{3}}{5} + 4, 1 \right\}, \frac{2}{5}(\sqrt{3} + 50)$
13.  $\{2, 10, 10\}, \{4\sqrt{3} - 5, 1\}, 8(4\sqrt{3} - 3)$
14.  $\{2, 10, 11\}, \left\{ 8\sqrt{2} - 3\sqrt{3} + 2\sqrt{6} - 4, \frac{8\sqrt{2}-2\sqrt{3}-6\sqrt{6}+5}{\sqrt{3}-2} \right\}, \frac{4(39\sqrt{2}-32\sqrt{3}+\sqrt{6}-56)}{\sqrt{3}-2}$
15.  $\{3, 3, 3\}, \{1, 1\}, 6\sqrt{2}$
16.  $\{3, 5, 5\}, \left\{ \frac{-16\sqrt{2}-2\sqrt{3}+9\sqrt{6}+4}{-416\sqrt{2}-464\sqrt{3}+240\sqrt{6}+804}, 1 \right\}, \frac{8(-925\sqrt{2}-228\sqrt{3}+534\sqrt{6}+395)}{201\sqrt{2}+120\sqrt{3}-116\sqrt{6}-208}$
17.  $\{3, 5, 6\}, \left\{ \frac{1}{47}(-7)(3\sqrt{3} - 11), \frac{2}{47}(485 - 269\sqrt{3}) \right\}, \frac{1}{47}(2983\sqrt{3} - 4906)$
18.  $\{3, 6, 6\}, \left\{ 1 + \frac{1}{\sqrt{2}}, 1 \right\}, \frac{1}{4}(17\sqrt{2} + 1)$
19.  $\{3, 6, 7\}, \left\{ \frac{1}{22}(64\sqrt{3} + 109), \frac{1}{44}(75\sqrt{3} + 131) \right\}, \frac{1}{44}(349\sqrt{3} + 499)$
20.  $\{3, 7, 7\}, \left\{ \frac{12\sqrt{2}-2\sqrt{3}-7\sqrt{6}+4}{-360\sqrt{2}-432\sqrt{3}+208\sqrt{6}+748}, 1 \right\}, \frac{8(-537\sqrt{2}+220\sqrt{3}+310\sqrt{6}-381)}{-187\sqrt{2}-104\sqrt{3}+108\sqrt{6}+180}$
21.  $\{3, 8, 8\}, \left\{ \frac{-27\sqrt{2}+6\sqrt{6}+54}{-4\sqrt{2}-12\sqrt{3}+12\sqrt{6}+8}, 1 \right\}, -\frac{4(-36\sqrt{2}-58\sqrt{3}+27\sqrt{6}+36)}{9(-\sqrt{2}-3\sqrt{3}+3\sqrt{6}+2)}$
22.  $\{3, 9, 9\}, \{2\sqrt{2} - 1, 1\}, 2(\sqrt{2} + 4)$
23.  $\{3, 9, 10\}, \left\{ \frac{3\sqrt{3}}{2} + 1, \frac{5\sqrt{3}}{2} - 1 \right\}, 32\sqrt{3} + 1$

24.  $\{3, 9, 11\}, \left\{ \frac{1}{2} (9\sqrt{2} + 7\sqrt{6} - 1), \frac{-7\sqrt{2} + \sqrt{3} + \sqrt{6} - 2}{2(\sqrt{3} - 2)} \right\},$   
 $480\sqrt{3} + 3\sqrt{\frac{3}{2}} - \frac{5}{\sqrt{2}} + 832$
25.  $\{3, 10, 10\}, \left\{ \frac{-2\sqrt{2} - 2\sqrt{3} + \sqrt{6}}{4(-\sqrt{2} - 2\sqrt{3} + \sqrt{6} + 1)}, 1 \right\}, \frac{4(\sqrt{2} + 4\sqrt{3} - 4\sqrt{6} - 6)}{-\sqrt{2} - 2\sqrt{3} + \sqrt{6} + 1}$
26.  $\{4, 4, 4\}, \{1, 1\}, \frac{8}{\sqrt{3}}$
27.  $\{4, 4, 5\}, \left\{ \frac{72\sqrt{2} - 56\sqrt{3} - 35\sqrt{6} + 84}{9(-269\sqrt{2} + 228\sqrt{3} + 156\sqrt{6} - 396)}, \frac{2\sqrt{3} - 14\sqrt{6} + 39}{9(-54\sqrt{2} + 43\sqrt{3} + 30\sqrt{6} - 70)} \right\},$   
 $\frac{8(-818532\sqrt{2} + 667879\sqrt{3} + 472644\sqrt{6} - 1156969)}{27(-80858\sqrt{2} + 65976\sqrt{3} + 46679\sqrt{6} - 114264)}$
28.  $\{4, 4, 6\}, \left\{ \frac{1}{333}(-8)(5\sqrt{3} - 36), \frac{1}{333}(-8)(5\sqrt{3} - 36) \right\}, -\frac{16(15\sqrt{3} - 256)}{999}$
29.  $\{4, 4, 7\}, \left\{ \frac{-72\sqrt{2} - 52\sqrt{3} + 37\sqrt{6} + 96}{9(-271\sqrt{2} - 228\sqrt{3} + 156\sqrt{6} + 396)}, \frac{-18\sqrt{2} - 2\sqrt{3} + 4\sqrt{6} + 33}{9(-54\sqrt{2} - 41\sqrt{3} + 30\sqrt{6} + 74)} \right\},$   
 $\frac{8(-864972\sqrt{2} - 705917\sqrt{3} + 499428\sqrt{6} + 1222595)}{27(-81146\sqrt{2} - 66216\sqrt{3} + 46847\sqrt{6} + 114696)}$
30.  $\{4, 5, 5\}, \left\{ \frac{289\sqrt{3} + 546}{1089}, 1 \right\}, \frac{8(3815\sqrt{3} - 6084)}{1089}$
31.  $\{4, 5, 6\}, \left\{ \frac{8(-20\sqrt{2} - 32\sqrt{3} + 11\sqrt{6} + 56)}{3(8\sqrt{2} - 7)(5\sqrt{3} - 9)}, -\frac{8(-159\sqrt{2} - 208\sqrt{3} + 92\sqrt{6} + 360)}{(8\sqrt{2} - 7)(5\sqrt{3} - 9)} \right\},$   
 $-\frac{16(-1456\sqrt{2} - 1253\sqrt{3} + 840\sqrt{6} + 2173)}{3(8\sqrt{2} - 7)(5\sqrt{3} - 9)}$
32.  $\{4, 5, 7\}, \left\{ \frac{3}{73}(61\sqrt{3} + 106), \frac{5}{73}(36\sqrt{3} + 47) \right\}, \frac{8}{657}(1199\sqrt{3} - 432)$
33.  $\{4, 6, 6\}, \left\{ \frac{3}{2}, 1 \right\}, \frac{337}{48\sqrt{3}}$
34.  $\{4, 6, 7\}, \left\{ \frac{14\sqrt{2} - 24\sqrt{3} - 9\sqrt{6} + 40}{351\sqrt{2} - 456\sqrt{3} - 204\sqrt{6} + 792}, \frac{-9\sqrt{2} + 16\sqrt{3} + 2\sqrt{6}}{8(-36\sqrt{2} + 19\sqrt{3} + 20\sqrt{6} - 30)} \right\},$   
 $\frac{148041\sqrt{2} - 122048\sqrt{3} - 85479\sqrt{6} + 211408}{28728\sqrt{2} - 26010\sqrt{3} - 16584\sqrt{6} + 45045}$
35.  $\{4, 7, 7\}, \left\{ \frac{829\sqrt{3} + 1374}{1503}, 1 \right\}, -\frac{8(4409\sqrt{3} - 8496)}{1503}$
36.  $\{4, 7, 8\}, \left\{ \frac{-55\sqrt{2} - 84\sqrt{3} + 31\sqrt{6} + 140}{-53\sqrt{2} - 84\sqrt{3} + 29\sqrt{6} + 148}, -\frac{3\sqrt{6}(\sqrt{3} - 1)}{-53\sqrt{2} - 84\sqrt{3} + 29\sqrt{6} + 148} \right\},$   
 $\frac{16(-6642\sqrt{2} - 2810\sqrt{3} + 3834\sqrt{6} + 4857)}{9(-53\sqrt{2} - 84\sqrt{3} + 29\sqrt{6} + 148)}$

37.  $\{4, 8, 8\}, \left\{\frac{5}{3}, 1\right\}, \frac{88}{9\sqrt{3}}$
38.  $\{4, 8, 9\}, \left\{\frac{1}{9}(9\sqrt{2} - 2\sqrt{3} + 2\sqrt{6} + 9), \frac{1}{9}(9\sqrt{2} + 2\sqrt{3} - 2\sqrt{6} + 9)\right\}, \frac{124\sqrt{2}}{27} + 8$
39.  $\{4, 8, 10\}, \left\{\frac{5}{9}(4\sqrt{3} + 3), \frac{1}{9}(16\sqrt{3} + 21)\right\}, 32\sqrt{3} + \frac{872}{27}$
40.  $\{4, 8, 11\}, \left\{\frac{1}{9}(48\sqrt{2} + 17\sqrt{3} + 26\sqrt{6} + 36), \frac{2\sqrt{3}+14\sqrt{6}+15}{18-9\sqrt{3}}\right\}, -\frac{4(521\sqrt{2}+864\sqrt{3}+335\sqrt{6}+1512)}{27(\sqrt{3}-2)}$
41.  $\{4, 9, 9\}, \left\{\frac{2}{99}(49\sqrt{3} - 36), 1\right\}, \frac{2}{99}(441 - 20\sqrt{3})$
42.  $\{4, 9, 10\}, \left\{\frac{-\sqrt{2}+2\sqrt{3}+2\sqrt{6}+1}{9\sqrt{2}-6\sqrt{3}-6\sqrt{6}+27}, \frac{3\sqrt{2}-8\sqrt{3}-2\sqrt{6}}{4(-3\sqrt{2}+2\sqrt{3}+2\sqrt{6}-9)}\right\}, \frac{4(16\sqrt{2}-4\sqrt{3}+3\sqrt{6}+20)}{9\sqrt{2}-6\sqrt{3}-6\sqrt{6}+27}$
43.  $\{5, 5, 5\}, \{1, 1\}, 12\sqrt{2}(3\sqrt{3} - 5)$
44.  $\{5, 5, 6\}, \left\{\frac{8}{11}(23\sqrt{3} - 38), \frac{8}{11}(23\sqrt{3} - 38)\right\}, \frac{1}{11}(-112)(91\sqrt{3} - 158)$
45.  $\{5, 5, 7\}, \left\{-\frac{1}{-10\sqrt{2}-8\sqrt{3}+6\sqrt{6}+13}, -\frac{1}{-10\sqrt{2}-8\sqrt{3}+6\sqrt{6}+13}\right\}, \frac{8(-104\sqrt{2}-73\sqrt{3}+60\sqrt{6}+127)}{-13\sqrt{2}-12\sqrt{3}+8\sqrt{6}+20}$
46.  $\{5, 6, 6\}, \left\{\frac{1}{5\sqrt{2}-4\sqrt{3}-3\sqrt{6}+8}, 1\right\}, 2\sqrt{2}(\sqrt{3} - 1)^3 \left(\frac{(\sqrt{3}+1)^6}{512(5\sqrt{2}-4\sqrt{3}-3\sqrt{6}+8)} + 2\right)$
47.  $\{5, 6, 7\}, \left\{\frac{1}{131}(112\sqrt{3} + 67), \frac{2}{131}(33\sqrt{3} + 56)\right\}, \frac{2}{131}(1208\sqrt{3} - 1757)$
48.  $\{5, 6, 8\}, \left\{\frac{9(-68\sqrt{2}-88\sqrt{3}+39\sqrt{6}+152)}{-27\sqrt{2}-40\sqrt{3}+14\sqrt{6}+72}, \frac{9(-40\sqrt{2}-21\sqrt{3}+24\sqrt{6}+37)}{4(-69\sqrt{2}-112\sqrt{3}+41\sqrt{6}+192)}\right\}, \frac{374744\sqrt{2}+310192\sqrt{3}-216362\sqrt{6}-537264}{-3296\sqrt{2}-3315\sqrt{3}+1904\sqrt{6}+5739}$
49.  $\{5, 7, 7\}, \{-10\sqrt{2} + 8\sqrt{3} + 6\sqrt{6} - 13, 1\}, 508\sqrt{2} - 480\sqrt{3} - 292\sqrt{6} + 832$
50.  $\{5, 7, 8\}, \left\{\frac{23}{\sqrt{3}} - \frac{34}{3}, \frac{1}{3}(-19)(\sqrt{3} - 2)\right\}, \frac{8}{27}(2110\sqrt{3} - 3633)$
51.  $\{5, 7, 9\}, \left\{\frac{-54\sqrt{2}-32\sqrt{3}+31\sqrt{6}+55}{\sqrt{3}-2}, \frac{50\sqrt{2}+28\sqrt{3}-29\sqrt{6}-49}{\sqrt{3}-2}\right\}, \frac{4(-385\sqrt{2}+2\sqrt{3}+222\sqrt{6}-4)}{\sqrt{3}-2}$
52.  $\{5, 7, 10\}, \{26 - 11\sqrt{3}, 19\sqrt{3} - 26\}, 208 - 56\sqrt{3}$
53.  $\{5, 7, 11\}, \left\{\frac{70\sqrt{2}-17\sqrt{3}-42\sqrt{6}+26}{\sqrt{3}-2}, 24\sqrt{2} - 8\sqrt{6} + 13\right\}, -\frac{4(-953\sqrt{2}+32\sqrt{3}+591\sqrt{6}+56)}{\sqrt{3}-2}$

54.  $\{5, 8, 8\}, \left\{ \frac{3843\sqrt{2}-3240\sqrt{3}-2217\sqrt{6}+5616}{109\sqrt{2}-84\sqrt{3}-61\sqrt{6}+148}, 1 \right\}, -\frac{16(-6822\sqrt{2}+5762\sqrt{3}+3942\sqrt{6}-9969)}{9(-109\sqrt{2}+84\sqrt{3}+61\sqrt{6}-148)}$
55.  $\{5, 8, 9\}, \left\{ \frac{22}{47} (7\sqrt{3} - 10), \frac{2}{423} (68\sqrt{3} + 111) \right\}, \frac{1}{423} (-8) (313\sqrt{3} - 897)$
56.  $\{5, 8, 10\}, \left\{ \frac{-855\sqrt{2}-1040\sqrt{3}+493\sqrt{6}+1800}{-93\sqrt{2}-148\sqrt{3}+53\sqrt{6}+252}, \frac{-91\sqrt{2}-20\sqrt{3}+51\sqrt{6}+28}{-279\sqrt{2}-444\sqrt{3}+159\sqrt{6}+756} \right\},$   
 $-\frac{16(-838\sqrt{2}-1555\sqrt{3}+486\sqrt{6}+2698)}{3(-93\sqrt{2}-148\sqrt{3}+53\sqrt{6}+252)}$
57.  $\{5, 9, 9\}, \left\{ \frac{832\sqrt{2}-654\sqrt{3}-480\sqrt{6}+1134}{-26\sqrt{2}+23\sqrt{3}+16\sqrt{6}-39}, 1 \right\}, -\frac{4(-1056\sqrt{2}+895\sqrt{3}+608\sqrt{6}-1554)}{-39\sqrt{2}+32\sqrt{3}+23\sqrt{6}-52}$
58.  $\{6, 6, 6\}, \{1, 1\}, 3$
59.  $\{6, 6, 7\}, \left\{ -5\sqrt{2} - 4\sqrt{3} + 3\sqrt{6} + 8, \frac{-\sqrt{2}+\sqrt{6}+4}{\sqrt{3}+2} \right\}, \frac{-2541\sqrt{2}-2048\sqrt{3}+1547\sqrt{6}+3584}{16\sqrt{3}+32}$
60.  $\{6, 6, 8\}, \{2, 2\}, \frac{283}{24\sqrt{3}}$
61.  $\{6, 6, 9\}, \left\{ \sqrt{2} + 2, \sqrt{2} + 2 \right\}, 8 + \frac{33}{2\sqrt{2}}$
62.  $\{6, 6, 10\}, \left\{ 2(\sqrt{3} + 2), 2(\sqrt{3} + 2) \right\}, 32\sqrt{3} + \frac{513}{8}$
63.  $\{6, 6, 11\}, \left\{ 5\sqrt{2} + 4\sqrt{3} + 3\sqrt{6} + 8, \frac{2(\sqrt{2}+\sqrt{6}+4)}{(\sqrt{3}-1)^2} \right\}, \frac{2541\sqrt{2}+2048\sqrt{3}+1547\sqrt{6}+3584}{32-16\sqrt{3}}$
64.  $\{6, 7, 7\}, \left\{ \frac{1}{584} (189\sqrt{3} + 338), 1 \right\}, \frac{1}{73} (-2) (624\sqrt{3} - 1205)$
65.  $\{6, 7, 8\}, \left\{ \frac{9(8\sqrt{2}+7)(3\sqrt{3}-5)}{8(-33\sqrt{2}+56\sqrt{3}+20\sqrt{6}-96)}, -\frac{9(-67\sqrt{2}+88\sqrt{3}+39\sqrt{6}-152)}{-27\sqrt{2}+40\sqrt{3}+13\sqrt{6}-72} \right\},$   
 $\frac{2(-90400\sqrt{2}+75160\sqrt{3}+52195\sqrt{6}-130176)}{-1592\sqrt{2}+1635\sqrt{3}+920\sqrt{6}-2829}$
66.  $\{6, 7, 9\}, \left\{ \frac{1}{478} (181\sqrt{3} + 173), \frac{1}{239} (-2) (384\sqrt{3} - 791) \right\}, \frac{1}{478} (7943 - 2855\sqrt{3})$
67.  $\{6, 7, 10\}, \left\{ \frac{-40\sqrt{2}-3\sqrt{3}+24\sqrt{6}+7}{-4\sqrt{2}-256\sqrt{3}+4\sqrt{6}+448}, \frac{-22\sqrt{2}-152\sqrt{3}+13\sqrt{6}+264}{\sqrt{2}+24\sqrt{3}-40} \right\},$   
 $\frac{-43906\sqrt{2}-157296\sqrt{3}+25344\sqrt{6}+272432}{-112\sqrt{2}-2625\sqrt{3}+64\sqrt{6}+4545}$
68.  $\{6, 8, 8\}, \left\{ \frac{1}{88} (-3) (5\sqrt{3} - 36), 1 \right\}, \frac{2}{99} (243 - 20\sqrt{3})$
69.  $\{6, 8, 9\}, \left\{ \frac{8\sqrt{3}+\sqrt{6}+\frac{9}{\sqrt{2}}}{4\sqrt{3}+8\sqrt{6}+9}, \frac{2(16\sqrt{2}+9\sqrt{3}+4)}{3(4\sqrt{3}+8\sqrt{6}+9)} \right\}, \frac{288\sqrt{2}+353\sqrt{3}+64\sqrt{6}+72}{36\sqrt{2}+27\sqrt{6}+144}$
70.  $\{7, 7, 7\}, \{1, 1\}, 12\sqrt{2} (3\sqrt{3} - 5)$

71.  $\{7, 7, 8\}, \left\{ \frac{3}{59} (451\sqrt{3} - 762), \frac{3}{59} (451\sqrt{3} - 762) \right\}, \frac{8}{59} (4094\sqrt{3} - 7063)$
72.  $\{7, 7, 9\}, \left\{ \frac{-402\sqrt{2}+240\sqrt{3}+232\sqrt{6}-416}{-2\sqrt{2}+9\sqrt{3}+\sqrt{6}-16}, \frac{-402\sqrt{2}+240\sqrt{3}+232\sqrt{6}-416}{-2\sqrt{2}+9\sqrt{3}+\sqrt{6}-16} \right\},$   
 $\frac{16(-925\sqrt{2}+228\sqrt{3}+534\sqrt{6}-395)}{-2\sqrt{2}+9\sqrt{3}+\sqrt{6}-16}$
73.  $\{7, 8, 8\}, \left\{ \frac{3825\sqrt{2}+3240\sqrt{3}-2211\sqrt{6}-5616}{-107\sqrt{2}-84\sqrt{3}+59\sqrt{6}+140}, 1 \right\}, \frac{16(-6462\sqrt{2}-5474\sqrt{3}+3726\sqrt{6}+9465)}{9(-107\sqrt{2}-84\sqrt{3}+59\sqrt{6}+140)}$

## 2 List of 35 triangles that cannot form Lagrange relative equilibrium

The format is

No.  $\{i, j, k\}, \{m_1/m_3, m_2/m_3\}$ .

1.  $\{1, 2, 2\}, \{-11.9582, 1\}$
2.  $\{1, 3, 3\}, \{-27.8385, 1\}$
3.  $\{1, 4, 4\}, \{-35.9773, 1\}$
4.  $\{1, 5, 5\}, \{-25.0661, 1\}$
5.  $\{2, 2, 3\}, \{-1.32406, -1.32406\}$
6.  $\{2, 3, 3\}, \{-2.8453, 1\}$
7.  $\{2, 3, 4\}, \{-2.04735, -0.960424\}$
8.  $\{2, 4, 4\}, \{-3.68469, 1\}$
9.  $\{2, 4, 5\}, \{-2.2913, -0.65559\}$
10.  $\{2, 5, 5\}, \{-1.93551, 1\}$
11.  $\{2, 5, 6\}, \{-1.09275, 0.0168562\}$
12.  $\{2, 6, 7\}, \{-11.9365, -4.26736\}$
13.  $\{2, 7, 8\}, \{-4.36437, -1.44109\}$
14.  $\{2, 8, 9\}, \{-2.81878, -1.08852\}$
15.  $\{2, 9, 10\}, \{-1.81165, -0.979754\}$
16.  $\{3, 3, 4\}, \{-0.409529, -0.409529\}$

17.  $\{3, 3, 5\}, \{-1.13981, -1.13981\}$
18.  $\{3, 4, 4\}, \{-0.178627, 1.\}$
19.  $\{3, 4, 5\}, \{-0.341506, 0.\}$
20.  $\{3, 4, 6\}, \{-0.727295, -0.401743\}$
21.  $\{3, 5, 7\}, \{-34.6882, -23.0504\}$
22.  $\{3, 6, 8\}, \{-2.38122, -1.35093\}$
23.  $\{3, 7, 8\}, \{-10.9952, -6.21539\}$
24.  $\{3, 7, 9\}, \{-1.48058, -0.866213\}$
25.  $\{3, 8, 9\}, \{-7.68811, -5.01608\}$
26.  $\{3, 8, 10\}, \{-0.93946, -0.654261\}$
27.  $\{4, 5, 8\}, \{-1.76306, -1.47596\}$
28.  $\{4, 6, 8\}, \{-4.70644, -3.70644\}$
29.  $\{4, 6, 9\}, \{-1.05275, -0.820155\}$
30.  $\{4, 7, 9\}, \{-3.11171, -2.54125\}$
31.  $\{4, 7, 10\}, \{-0.657545, -0.539198\}$
32.  $\{5, 5, 8\}, \{-3.72751, -3.72751\}$
33.  $\{5, 5, 9\}, \{-0.875196, -0.875196\}$
34.  $\{5, 6, 9\}, \{-2.35604, -2.21278\}$
35.  $\{5, 6, 10\}, \{-0.542071, -0.508683\}$