Proof. Let

$$S(t) = \frac{\Delta(t)}{\sqrt{I(t)}}.$$

Then

$$\begin{aligned} \frac{d^2S}{dt^2} &= -\left\{ \sum_{i<j} (m_i + m_j) r_{ij}^{\alpha-2} + \frac{2K}{I} + \frac{1}{2I}\frac{d^2I}{dt^2} - \frac{3}{4I^2}\left(\frac{dI}{dt}\right)^2 \right\} S \\ &= -\omega^2 S, \end{aligned}$$

$$\omega^2 \geq \omega_0^2 > 0 \text{ where } \omega_0^2 = M\left(\frac{m_{min}^2}{MI_{max}}\right)^{(2-\alpha)/2} \quad \text{for } \alpha \leq 2.$$

□