Abstract

Jan. 23 (Mon.)

13:20 – 14:20 **Takao Komatsu** (Zhejiang Sci-Tech University)

Title: Arithmetic approach to p-numerical semigroups

Abstract: The p-numerical semigroup is a generalization of the algebraically defined numerical semigroup by focusing on the number of solutions of the corresponding linear diophantine equation. Numerical semigroups have been studied from various angles, such as algebraic curves and commutative algebra, but in this talk I will give their properties from the number-theoretic side.

14:40 – 15:40 **Koji Tasaka** (Aichi Prefectural University)

Title: Spherical designs and modular forms of the D_4 root lattice

Abstract: We will talk about a spherical design, one of important combinatorial structures, of the shells of the D4 lattice and its application to the theory of modular forms and Hecke operators on the space of harmonic polynomials. This is a joint work with Masatake Hirao and Hiroshi Nozaki.

16:00 – 17:00 **Kohta Gejima** (Osaka Metropolitan University)

Title: Determining cusp forms by critical values of Rankin-Selberg L-functions

Abstract: In this talk I will describe that a holomorphic cusp form g is uniquely determined by certain critical values of the family of Rankin–Selberg L-functions $L(f \otimes g, s)$, where f runs over a fixed orthogonal basis of the space of cusp forms of weight k. The ideas employed are simpler than those used in the previous results for the central value.

Jan. 24 (Tue.)

9:20 – 10:20 (EST 19:20 – 20:20, Jan. 23 (Mon.)) **Peter Humphries** (University of Virginia)

Title: Newform Theory

Abstract: When do there exist automorphic forms that are invariant under distinguished congruence subgroups? When is a period integral of automorphic forms exactly equal to an L-function? How can one quantify the complexity of an automorphic representation? We shall discuss three interrelated notions in the theory of automorphic forms and automorphic representations: newforms, L-functions, and conductors. In particular, we cover how to define the newform associated to an automorphic representation of GL_n, how to realise certain L-functions as period integrals involving newforms, and how to quantify the ramification of an automorphic representation in terms of properties of the newform. A key emphasis is the union of approaches to defining newforms in both nonarchimedean and archimedean settings. Finally, we will briefly discuss notions of newforms for groups other than GL_n.

10:40 – 11:40 **Eric Stade** (University of Colorado Boulder)

Title: An asymptotic orthogonality relation for $GL(n, \mathbb{R})$ and recurrence relations for Mellin transforms of $GL(n, \mathbb{R})$ Whittaker functions

Abstract: We discuss joint work with Dorian Goldfeld and Michael Woodbury on an asymptotic orthogonality relation, with a power-savings error term, for Fourier coefficients of $GL(n, \mathbb{R})$ Maass cusp forms.

Our proof of this orthogonality relation relies on two conjectures, one concerning lower bounds for certain Rankin-Selberg L-functions, and the other concerning recurrence relations for Mellin transforms of $GL(n, \mathbb{R})$ Whittaker functions. We further discuss joint work with Taku Ishii towards the second conjecture.

13:20 – 14:20 **Kenichi Namikawa** (Tokyo Denki University)

Title: An integrality of critical values of Rankin-Selberg L-functions for $\mathrm{GL}(n) \times \mathrm{GL}(n-1)$

Abstract: By using the generalized modular symbol method due to Kazhdan-Mazur-Schmidt, Raghuram proved the algebraicity of ratios of period integrals of Rankin-Selberg L-functions for $\mathrm{GL}(n) \times \mathrm{GL}(n-1)$ to Whittaker periods. In this talk, we consider a refinement of the generalized modular symbol method and we prove the algebraicity of critical values of Rankin-Selberg L-functions for $\mathrm{GL}(n) \times \mathrm{GL}(n-1)$ if the base fields are totally imaginary. Key ingredients of the proof are a description of local systems on $\mathrm{GL}(n)$ by Gel'fand-Tsetlin basis and an explicit formula for the archimedean local zeta integrals. Furthermore, we also propose a formulation for p-integral properties of these critical values. This is a joint work with Takashi Hara (Tsuda university) and Tadashi Miyazaki (Kitasato university).

14:40 – 15:40 **Kazuki Morimoto** (Kobe University)

Title: On Ichino-Ikeda type formula of Bessel periods for $(\mathrm{U}(2n), \mathrm{U}(1))$ and $(\mathrm{GL}(2n), \mathrm{GL}(1))$

Abstract: Under certain assumptions at archimedean places, we prove Ichino-Ikeda type formula of Bessel periods for $(\mathrm{U}(2n), \mathrm{U}(1))$ conjectured by Y. Liu. We prove it by combining theta lifts from $\mathrm{U}(2n)$ to $\mathrm{U}(2n)$ and Ichino-Ikeda type formula of Whittaker periods for $\mathrm{U}(2n)$. In a similar argument, we also show Ichino-Ikeda type formula of Bessel periods for $(\mathrm{GL}(2n), \mathrm{GL}(1))$ for any irreducible cuspidal tempered automorphic representations. This talk is based on a joint work with Masaaki Furusawa.

Jan. 25 (Wed.)

9:20 – 10:20 **Yugo Takanashi** (University of Tokyo)

Title: Parity of conjugate self-dual representations of inner forms of GL_n over p-adic fields

Abstract: There are two parametrizations of discrete series representations of GL_n over p-adic fields. One is the local Langlands correspondence, and the other is the local Jacquet-Langlands correspondence. The composite of these two maps the discrete series representations of an inner form of GL_n to Galois representations called discrete L-parameters. On the other hand, we can define the parity for each self-dual representation depending on whether the representation is orthogonal or symplectic. The composite preserves the notion of self-duality, and it transforms the parity in a nontrivial manner. Prasad and Ramakrishnan computed the transformation law, and Mieda proved its conjugate self-dual analog under some conditions on groups and representations. We will talk about the proof of the general case of this analog. We use the globalization method, as in the proof of Prasad and Ramakrishnan.

10:40 – 11:40 **Shuji Horinaga** (Nippon telegraph and telephone corporation)

Title: Cuspidal components of Siegel modular forms for large discrete series representations of $\mathrm{Sp}_4(\mathbb{R})$

Abstract: As far as I know, there are no known results of an explicit description of cuspidal components for non-holomorphic automorphic forms except for nearly holomorphic modular forms. In this talk, we investigate the cuspidal components and the structures for automorphic forms on Sp_4 which generate large discrete series representations by the explicit formulas of degenerate Whittaker functions of large discrete series representations. This talk is based on the joint work with Hiro-aki Narita.

13:20 – 14:20 **Takuya Yamauchi** (Tohoku University)

Title: Equidistribution theorems for holomorphic Siegel cusp forms of general degree: the level aspect

Abstract: In this talk, I will explain my recent work with Kim and Wakatsuki on equidistribution theorems for holomorphic Siegel cusp forms of general degree in the level aspect.

14:40 – 15:40 **Kazunari Sugiyama** (Chiba Institute of Technology)

Title: The modularity of Siegel's zeta functions

Abstract: Siegel defined zeta functions associated with indefinite quadratic forms, and proved their analytic properties such as analytic continuations and functional equations. Coefficients of these zeta functions are called measures of representations, and play an important role in the arithmetic theory of quadratic forms. In a 1938 paper, Siegel made a comment to the effect that the modularity of his zeta functions would be proved by a suitable converse theorem. Later in a 1951 paper, Siegel proved that the measures of representations appear as Fourier coefficients of some real analytic automorphic forms. Further, an explicit formula for Siegel's zeta functions is proved by Ibukiyama. On the other hand, it is only recently that papers on Weil-type converse theorems for Maass forms have appeared. The purpose of the present talk is to accomplish Siegel's original plan by using a new converse theorem. It is also shown that "half" of Siegel's zeta functions correspond to holomorphic modular forms.

16:00 – 17:00 **Masao Tsuzuki** (Sophia University)

Title: A relative trace formula on $\mathrm{GL}(n)$ and its application

Abstract: The Whittaker functional and the Rankin-Selberg integral are two of the most important and basic period integrals for automorphic forms on general linear groups. Taking the rational number field for the base field, we develop a version of relative trace formulas on $\mathrm{GL}(n)$, whose spectral side involves two of these period integrals with the minimal Eisenstein series being placed as the $\mathrm{GL}(n-1)$ integrand of the Rankin-Selberg period. On the geometric side, we observe that a simplication is brought by choosing test functions with sufficiently large mirabolic levels. If time permits, as an application, we shall report a new non-vanishing result for central standard L-values for a family of $\mathrm{GL}(n)$ Hecke Maass cusp forms with growing prime mirabolic levels and nontrivial central characters.

Jan. 26 (Thu.)

9:20 – 10:20 **Shingo Sugiyama** (Nihon University)

Title: Resolvent trace formulas of Hecke operators and optimal estimates of the Hurwitz class numbers

Abstract: The Eichler-Selberg trace formula for a Hecke operator has many applications to dimension formulas and equidistributions of Hecke eigenvalues. In this talk, we give a resolvent trace formula of a Hecke operator. We also give an application of the resolvent trace formula to optimal estimates of an average of the Kronecker-Hurwitz class numbers. This is a joint work with Masao Tsuzuki (Sophia University).

10:40 – 11:40 **Yoshinori Mizuno** (Tokushima University)

Title: On Hutchinson's conjecture

Abstract: Let $j(\tau)$ be the elliptic modular function. Following Gross and Zagier, we define

$$J(d_1, d_2) := \left(\prod_{[\tau_1], \mathrm{disc}(\tau_1)=d_1} \prod_{[\tau_2], \mathrm{disc}(\tau_2)=d_2} (j(\tau_1) - j(\tau_2)) \right)^{\frac{4}{w_1 w_2}},$$

where d_i is a negative discriminant, w_i is the number of units in the quadratic order $\mathcal{O}_{d_i}$ and the product is taken over all quadratic irrationals τ_i of discriminant d_i in the upper-half plane modulo the action of $\mathrm{SL}_2(\mathbb{Z})$. In 1985, Gross and Zagier established a closed formula of $J(d_1, d_2)^2$ when d_1, d_2 are relatively prime negative fundamental discriminants. In 1998, Tim Hutchinson presented a conjectual extension of the closed formula to the case when d_1 and d_2 are not necessarily fundamental, and $\gcd(d_1, d_2)$ is a power of a prime not dividing the product of the conductors of d_1 and d_2. In this talk, we give a proof of this conjecture when d_1, d_2 are fundamental and $\gcd(d_1, d_2)$ is a power of a prime. The proof proceeds along the lines of the second proof given in Gross-Zagier's paper. But to do so, we need to study the class number $h(d_1, d_2, \delta)$ of pairs of positive-definite binary quadratic

forms of discriminant d_1, d_2 with codiscriminant δ, when $\gcd(d_1, d_2)$ is a power of a prime and δ is arbitrary. We give a closed formula of the number $h(d_1, d_2, \delta)$ and relate it to the Fourier coefficients of Hilbert-Eisenstein series of weight 1 associated to the quadratic order $\mathcal{O}_{d_1 d_2}$. Ideal theory of $\mathcal{O}_{d_1 d_2}$ and the genus character $\chi_{d_1,d_2}^{(d_1 d_2)}$ (of non-fundamental discriminant $d_1 d_2$) together with an explicit formula of the L-function of $\chi_{d_1,d_2}^{(d_1 d_2)}$ are indispensable.

13:20 – 14:20 **Shotaro Kimura** (Waseda University)

Title: On constructions of modular differential equations

Abstract: A modular differential equation is a differential equation whose solution space satisfies modular invariance. The studies of modular differential equations appear in a variety of fields beyond number theory. For example, it has applications to vertex operator algebras, conformal field theory, elliptic genera, and so on. Thus it is an interesting problem to generalize modular differential equations to various modular forms. In this talk, we introduce the construction and properties of the modular differential equation for skew-holomorphic Jacobi forms which are not holomorphic. Moreover, we show a unified construction method of higher-order modular differential equations for several modular forms by Rankin-Cohen brackets.

14:40 – 15:40 **Siegfried Böcherer** (University of Mannheim)

Title: On equivariant holomorphic differential operators starting from vector-valued cases

Abstract: The theory of Rankin-Cohen bilinear holomorphic differential operators is well explored for scalar-valued cases, mainly by the work of Ibukiyama. Not so much is known when we start from vector-valued automorphy factors. We will describe some constructions starting from nonholomorphic operators of Maaß-Shimura type. We focus on operators of order one, but by some compatibility with tensor products we can cover more general situations. For the case of symmetric tensor representations we can however give quite complete results by a direct approach. Some parts of the talk are based on the Mannheim PhD-thesis 2021 by Julia Meister.

Jan. 27 (Fri.)

9:20 – 10:20 **Keiichi Gunji** (Chiba Institute of Technology)

Title: Simple functional equations satisfied by Siegel Eisenstein series of level p

Abstract: Let p be an odd prime number. We consider the functional equations of the Siege Eisenstein series of level p with trivial or quadratic character. As is well-known, for degree n case, the dimension of the space of Siege Eisenstein series is $(n+1)$-dimensional, thus the functions equations are written by matrix form. This matrix seems to become complicated, however we can choose simple equations by considering the $U(p)$-eigen functions. In this talk we first study the $U(p)$-action on the space of Siegel Eisenstein series to get the $U(p)$-eigen functions. Next we shall give the explicit form of the functional equations for eigen functions, as a consequence we explain that one can write the representative matrix of the functional equations by the products of rather simple matrices.

10:40 – 11:40 **Tomoyoshi Ibukiyama** (Osaka University)

Title: Differential operators on Siegel modular forms and Laplace transform

Abstract: The talk has two aims. One is to give a new special basis of the polynomial ring $\mathbb{C}[T]$ in components of $n \times n$ symmetric matrix T. The basis $P_\nu(T)$ is characterized by the action of $P_\nu(\partial_Z)$ on automorphy factors $\det(CZ + D)^s$, where $\partial_Z = \left(\frac{\partial}{\partial z_{ij}}\right)$ for the vaiable Z of the Siegel upper half space H_n. (Our theory is quite different from the well-known theory by Shimura.) The basis is given as coefficients of certain explicitly described generating series. The second aim is to apply this to the so-called pullback formula of the Siegel Eisenstein series of degree $2n$ and determine all necessary constants appearing there.